XIV. *Account of Pendulum Experiments undertaken in the Harton Colliery, for the purpose of determining the Mean Density of the Earth.*

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Section I.—*Introductory and Historical.*

1. In the spring of the year 1826, the idea occurred to me of attempting a determination of the Mean Density of the Earth by means of pendulum-experiments at the top and the bottom of a deep mine; and rough preliminary calculations, which seemed to show that the effect of accidental errors of observation on the ultimate result would probably be less than in the methods which up to that time had been used, confirmed me in the wish to try it. The nature of these preliminary calculations was nearly the following.

2. Conceive the earth to be a sphere of radius R and mean density D, surrounded by a spherical shell of thickness c and density d, so that the radius of the external surface is $R+c$. As the attraction of the spherical shell upon a point at or within its inner surface is nothing, the attraction at the confines of the inner sphere and shell is represented by $\frac{4\pi}{3}\cdot\frac{R^3D}{R^2}=\frac{4\pi}{3}RD$; and the attraction at the external surface of the shell is represented by $\frac{4\pi}{3}\cdot\frac{R^3D+(\overline{R+c}^3-R^3)d}{(R+c)^2}$, which to the first power of $\frac{c}{R}$ is $\frac{4\pi}{3}RD\left(1-\frac{2c}{R}+\frac{3c}{R}\cdot\frac{d}{D}\right)$. Calling the gravity at those two points respectively G and g, $\frac{g}{G}=1-\frac{2c}{R}+\frac{3c}{R}\cdot\frac{d}{D}$; from which $\frac{d}{D}=\frac{R}{3c}\cdot\frac{g}{G}-\left(\frac{R}{3c}-\frac{2}{3}\right)$. Considering the fractions $\frac{d}{D}$ and $\frac{g}{G}$ as the only quantities in this equation liable to sensible numerical error, $\delta\left(\frac{d}{D}\right)=\frac{R}{3c}\delta\left(\frac{g}{G}\right)$. As the density of the shell, or d, may be ascertained by actual examination, $\delta\left(\frac{d}{D}\right)=-\frac{d}{D}\cdot\frac{\delta D}{D}$; and therefore $\frac{\delta D}{D}=-\frac{D}{d}\cdot\frac{R}{3c}\,\delta\,\frac{g}{G}$.

3. In order to give a numerical value to the error of result, suppose that $\frac{D}{d}=2$ nearly (a proportion rather higher than Maskelyne's); that $\frac{R}{c}=16000$ (which supposes the thickness of the shell, or the depth of the mine, to be a quarter of a mile nearly); and that $\delta\left(\frac{g}{G}\right)=\frac{1}{432000}$ (which corresponds to an error of 0·1 per day in the

vibrations of a seconds' pendulum). Then $\frac{\delta D}{D} = -2 \cdot \frac{16000}{3} \cdot \frac{1}{432000} = -\frac{2}{81}$: or the mean density would be determined with an error not exceeding $\frac{1}{40}$th part of the whole. This error, I apprehend, is far less than those to which CAVENDISH's experiments were liable (REICH's and BAILY's experiments had not been made at the date of this investigation, but I do not except them from the same remark). It is also quite as small as that to which the mere astronomical determination in the Schehallien experiment was subject. When other elements of calculation are examined, as the general simplicity in the form of the ground for the mine-experiment and the complexity for the mountain-experiment, the accurate knowledge of the geology for the mine-experiment and the obscurity for the mountain-experiment, the difference becomes still more striking in favour of the mine-experiment as compared with that on Schehallien.

4. In a subsequent section, means will be given for enabling the reader to judge whether an accuracy like 0·1 vibration per day in the difference of the rates of pendulums above and below, in the mine-experiment, has really been obtained. Methods will also be indicated for computing the corrections depending on the irregularities of the earth's surface. I advert to these at present only to have the opportunity of explaining that the form of computation exhibited above is not final, but is merely intended for a preliminary calculation, showing the antecedent plausibility of the experiment.

5. Upon communicating my views to Mr. (now Dr.) WHEWELL (then, like myself, a resident Fellow of Trinity College, Cambridge), I found him entirely disposed to join me in undertaking the experiment. My first idea had been, to ascertain the rate of a clock at the top and the bottom of a mine; and the locality which first occurred to me was the Ecton Mine in Staffordshire. But it was soon settled, on discussion with Mr. WHEWELL, that detached pendulums would be preferable to clock pendulums; and, the Ecton Mine having proved on examination to be ill suited to our purpose, the Dolcoath Mine near Camborne in Cornwall was selected (partly at the suggestion of JOHN TAYLOR, Esq.). The Royal Society and the Board of Longitude most liberally lent to us invariable pendulums, clocks, and other apparatus, sufficient for the simultaneous observation of a pendulum above and one below; and the Board of Admiralty lent us box and pocket chronometers, to be used principally for the comparison of the upper and lower clocks.

6. About the end of May 1826 we proceeded to Dolcoath, assisted by the friendly introductions of the late DAVIES GILBERT, Esq. and of Dr. PARIS; and we received from the resident authorities of the mine every possible assistance to our experiment; and from the late Lord DE DUNSTANVILLE, E. W. W. PENDARVES, Esq., and other gentlemen of the country the most hospitable attention to our personal comforts. And, in spite of the labours and misfortunes of the Cornish enterprises, I do not doubt that they are regarded by my companion (as well as by myself) as among the most

interesting in our lives. Stations were provided for us at the surface and near the lowest part of the mine; the latter being at a short distance from the South Shaft or Harriet Shaft, near the junction of the granite and the killas, in the 180-fathom level (the depth being measured from the adit for the discharge of the water pumped from the mine), or nearly 1200 feet below the surface. This place, as is usual in the Cornish mines, could only be reached by ladders.

7. Our intention was, to compare the vibrations of a detached pendulum in each station with the vibrations of a clock pendulum, in the manner which has acquired so much celebrity from the labours of KATER and SABINE, and to compare the two clocks by means of chronometers carried on the person of an attendant. One set of observations, extending over several hours (including if necessary more than one series of coincidences), was to be observed each day. When this should be carried far enough, the pendulums were to be reversed and the observations were to be repeated, or the two pendulums were to be compared, as might seem best. After overcoming some difficulties, and with very great personal labour, we found on computing approximately the results that the chronometer-comparisons were not trustworthy. We resumed the work, with a modification of the method of using the chronometers which promised to render the results more accurate. We were raising the lower pendulum up the South Shaft for the purpose of interchanging the two pendulums, when (from causes of which we are yet ignorant) the straw in which the pendulum-box was packed took fire, the lashings were burnt away, and the pendulum with some other apparatus fell to the bottom. This terminated our operations of 1826.

8. In the summer of 1828 we again attempted the experiment, in the same localities, and with the same general instrumental means. Our personal powers however were far greater than in 1826. We had now the assistance of the Rev. R. SHEEPSHANKS and of two junior observers (my brother Mr., now the Rev. W. AIRY, and the late Rev. S. JACKSON). Our plan, principally at the instance of Mr. SHEEPSHANKS, was so modified as to admit of incessant observations being made, day and night, for several days consecutively; and this arrangement greatly diminished the injurious effects of chronometer-errors. A new difficulty now presented itself in the irregular and varying form of the pendulums' knife-edges. After tedious experiments on these, which seemed at last likely to be successful, our labours were suddenly stopped by the occurrence of a "fall" in the mine. The lodes or metalliferous veins in the Cornish mines are usually bounded by nearly parallel planes inclined perhaps 30° or 40° to the vertical, and the removal of the vein-stuff (even when, as in this case, the vacuity has been filled up as much as circumstances permit) endangers the falling of the rock which is on the upper side of the lode. On this occasion, the general fall (in consequence of the precautions above described) did not exceed a few inches; but large masses of rock were detached from their places, and interrupted the working of the pumps; and our lower station was soon flooded by the rise of the water. And thus for the second time our attempts were defeated.

9. Many years passed on before sufficient leisure or sufficient motive for again trying the experiment presented itself to me. The only great improvement in the application of the pendulum to the measure of gravity was BESSEL's discovery, reduced to a practical form (as regarded the English construction of the pendulum) by Colonel SABINE, of the necessity of an increased correction for the pendulum's buoyancy in the atmosphere. I had, however, opportunities of observing the difficulties inherent in the CAVENDISH experiment, from witnessing the repetition of that experiment by my late friend Mr. F. BAILY. At length, in the year 1854, a new power was placed in my hands. The galvanic system was established at the Royal Observatory, and in the familiarity which we now possessed with telegraphic applications I perceived that the difficulty of comparing the upper and lower clocks would be almost entirely removed. The coal-mines of the Durham coal-field had been worked much deeper, and the facility of access to these mines would materially diminish the labour of the experiment; while the intimate acquaintance with the geological character of the country possessed by the coal-owners, and the general regularity of the beds, would give great confidence in the ultimate calculations of the attraction of the mass of matter principally affecting the experiment. After a lapse of twenty-six years from the last attempt in Cornwall, I therefore seriously took up the subject again, and proceeded personally to examine the fitness of the coal-mines of Durham for the experiment.

10. Assisted by the introductions of DAVID LIETCH, Esq., M.D., and by the local knowledge of JAMES MATHER, Esq., I had little difficulty in fixing on a mine. The deepest mines in the Durham coal-field are near the coast. The deepest of all is the Monk Wearmouth Colliery; but it is so close to the sea (its workings in fact extending far under the sea), that it seemed probable that more of disadvantage would be introduced by the complication of the elements of final computations than of advantage by its extreme depth. The next in depth, I believe, is the Harton Pit, at the distance of somewhat more than two miles from South Shields, and about the same distance from the coast. The general circumstances of form of surface, &c. are as favourable as can usually be found. Its depth is reputed to be 1260 feet. On making known to WILLIAM ANDERSON, Esq., the principal owner of the coal-mine, my wish to try the experiment in that place, I was at once assured that every assistance should be given to me. In company with C. W. ANDERSON, Esq. and with G. W. ARKLEY, Esq. (local viewer and superintendent of the mine-works) in 1854, August 5, I examined the buildings on the surface and the workings underground in the "Bensham seam," and stations for the pendulums were speedily chosen. The two stations were exactly in the same vertical. The upper station was a stable near the Mine Office. The lower station was a wide part of a gallery (now disused), less than 200 yards from the bottom of the shaft: in going from the bottom of the shaft to the station, one-half (roughly speaking) of the way passed along one of the tram-ways of the mine, and the other half was along the disused gallery; and at this distance the sound of

the coal-trams, &c. on the tram-way was inaudible. The Harton shaft is the downcast air-shaft for a very extensive series of workings, of which the upcast shaft is at the St. Hilda pit in the town of South Shields; and the lower pendulum station, from its proximity to the Harton pit, was constantly and agreeably ventilated with pure fresh air.

The owners of the mine immediately proceeded, at their own expense, to make the following preparations for the pendulums.

11. In the stable for the upper station, an additional brick wall was built round three sides of the space (about 16 feet square) intended for the pendulum room, and an additional substantial ceiling was constructed above it; and an anteroom was constructed on the fourth side; so that the room was in fact surrounded by double walls and ceiling. The immediate entrance from the external air was into the anteroom; and from the anteroom there were doors into the pendulum room and into a narrow room at one side. [In this narrow room were afterwards placed the galvanic batteries and journeyman-clock, and the lamp for illuminating the pendulum disk. The lamp for illuminating the face of the principal clock was in the anteroom, giving its light through a hole in the dividing wall; and the observer was stationed in the anteroom, his observing telescope being fixed in a hole in the wall.] From the gas-works of the mine, gas was led to a gas-stove in the anteroom, and also to a writing-lamp. In order to make the support of the pendulum firm, the soft earth was removed to the depth of 3 feet (at which level the hard clay was reached, which extends about 90 feet lower to the first bed of rock), and the cavity was filled with ashlar stones united with mortar at the joints: the surface was paved level with flagstones and bricks.

12. In the lower station, some upright shores which had been placed in the space assigned for the pendulum room were removed, and inclined rafters were substituted. The rock was cut level, and the bricks and flagstones were laid immediately on it. Brick walls were built enclosing a trapezoidal room nearly representing a square of 16 feet, with an anteroom, and a third room; and level ceilings were constructed above them.

13. For the galvanic communication between the two stations, two wires covered with gutta percha were led underground from the stable to the mouth of the pit; and were then carried down to the bottom of the pit, being supported in the following manner. The shaft is divided from top to bottom by a wooden partition or brattice. At every 100 feet in the descent, a wooden peg about 6 inches long and 2 inches in diameter was fixed in the brattice. The galvanic wires were wrapped round this peg, and were pinched fast by a piece of wood which was screwed to the peg. In this manner the wires were held perfectly steady, and no part was strained by an excessive weight hanging upon them. On reaching the bottom of the shaft, the wires were carried horizontally along the top of the galleries, protected by boards of wood, till they entered the lower station. The labour of Telegraph Engineers, which was

required for fixing these wires and for other adjustments of the galvanic apparatus, was contributed gratuitously by the Electric Telegraph Company; the work being done under the direction of their resident agent at Newcastle, Mr. MATHEWS.

14. The local arrangements being thus brought to a very satisfactory state, I had next to provide the proper apparatus. On my request, through Colonel SABINE, Treasurer of the Royal Society, the Council of the Society immediately placed at my disposal the instruments which they possessed and which applied to my wants. These were two invariable pendulums, both by THOMAS JONES (distinguished by the marks "1821" and "No. 8" respectively), which were preserved in the Kew Observatory; iron stands for the pendulums, accompanied by wooden stands for the comparison clocks, preserved in the Magnetic Office at Woolwich; and a clock by SHELTON (which I had formerly used in Cornwall, and which still bore my paper marks upon its pendulum). To these I added a clock by EARNSHAW (from the east dome of the Royal Observatory); a journeyman-clock (from the Royal Observatory) to be used in signal-transmission; two galvanometers, similar to the ordinary speaking-needle of the galvanic telegraph; galvanic batteries; barometers, thermometers, &c. Every instrument was brought, for trial, to the Royal Observatory.

15. Each iron stand consists of a horizontal triangle of iron bars for base; two inclined iron bars in a vertical plane for front, their tops being screwed to the sides of an iron box-frame which supports the agate-frame on which the knife-edge vibrates; and one inclined bent iron bar behind, screwed to the back of the same box-frame. On erecting one of the iron stands at the Royal Observatory, it was found impossible to pass the upper brass block of the pendulums (to which the steel knife-edge is screwed) through the holes of the box-frame. It was necessary therefore to enlarge the holes by filing; and as this weakened the front of the box-frame, the following apparatus was introduced in order to strengthen it. A small hexagonal iron frame was prepared, consisting of a horizontal base nearly equal in length to the interior measure of the box-frame, two vertical sides nearly half the height of the box-frame, two inclined sides, and a horizontal top about one-third as long as the bottom. Screws were tapped through the horizontal base. This apparatus was placed in the front opening of the box-frame; and, when the screws were driven, the short horizontal top was forced up to the upper bar of the box-frame so as to give it all desirable firmness. As the weakened stand, furnished with this apparatus, was evidently firmer than the other stand, a similar apparatus was provided for the other stand.

16. The agate-planes, and the knife-edges (which were much injured by rust), were reground and polished by Mr. SIMMS with the utmost care; the knife-edges being finished to an angle of about 120°. On mounting them at the Royal Observatory, it was found that they touched the agate-planes only in a few points; and the singularity of the mode of contact reminded me so strongly of the contact in Cornwall in 1828, that I do not doubt that one pendulum was the same and affected by the same faults. A few interchanges, &c. of pendulums showed that the fault was not in the agate-

planes, but in the knife-edges. Mr. SIMMS on examination found that, when the attaching screws were relaxed, the bearing on the agate-planes was continuous and perfect. It was evident therefore that the fault was in the surface of the brass blocks which carry the knife-edges. On filing these it was found easy to bend the knife-edge into any form. A surface was at length given to the brass which made the bearing of the knife-edges upon the agates absolutely perfect, as far as the eye could discover.

The upper and lower pendulums, when mounted in their proper stations, vibrated in parallel planes, as nearly as possible in the direction of magnetic East and West.

The graduated arcs for measuring the extent of vibration were divided to inches, with continuous numeration from one end; and were placed behind the pendulum tails.

The thermometers (two for each pendulum) were suspended in front, at about $\frac{1}{4}$ and $\frac{3}{4}$ the distances from the knife-edge to the bob. Their indications were found to be all sensibly accordant.

17. In fitting up the comparison-clocks, a small alteration was made which proved exceedingly convenient. The illuminated disk (to be concealed by the tail of the invariable pendulum) was an inclined section of a small cylindrical block attached by a central screw to the bob of the clock pendulum. The inclined surface was covered with gold-leaf. A hole (covered with glass) was made in each side of the clock-case, and through either of these the light of the illuminating lamp was thrown upon the gold-leaf. By shifting the lamp and turning the small cylindrical block, a brilliant light was reflected to the observing-telescope, the lamp being always in a distant position and in a lateral direction. Moreover, by slightly turning the clock stand in azimuth, the apparent breadth of the inclined disk was altered, and it could thus be adapted to disappearance behind the pendulum-tail.

The adjustible aperture through which the disk was seen, and which was covered by the pendulum-tail in its quiescent position, was in front of the clock-case.

18. By the side of each clock-face a galvanometer was fixed. The galvanic wires were led to and from the terminals of the galvanometer, not immediately, but through the intermediation of a circuit-breaker; so that the observer could at any time interrupt the current.

19. The journeyman-clock was thus fitted up. Two wires were led into it (one from the galvanic battery, and the other in continuation of the course of the same wire from the journeyman to the next comparison-clock), terminating within the journeyman in a pair of springs which performed the duty of circuit-breaker. Upon the minute-wheel of the journeyman were four pins, which, as the wheel revolved, pressed the two springs together, thus completing the circuit (in that part) at every 15^s of the journeyman's time.

20. The battery was the ordinary sand-battery, of 24 cells. The batteries and journeyman were in the side-room of the upper station.

The wire from one end of the battery was led to one terminal of the journeyman. From the other terminal of the journeyman a wire was led to the circuit-breaker in connexion with the galvanometer of the upper clock. From the other terminal of the galvanometer a wire was led down the mine-shaft to the circuit-breaker in connexion with the galvanometer of the lower clock. From the other terminal of this lower galvanometer a wire was led up the mine-shaft to the other pole of the battery. Thus when both the upper and the lower circuit-breakers completed contact (and at no other time), the journeyman-clock made the circuit complete and sent a current through both the galvanometers at every 15^s of the journeyman's time.

20*. Plate XI. contains views of the pendulum-apparatus nearly in the state in which it was used in the upper station. The principal diagram is a front view of the apparatus as mounted at Greenwich, taken with the camera lucida, and may be trusted for general accuracy. The iron bars of the pendulum-stand are about $1\frac{1}{4}$ inch square. The stand of the clock does not touch the pendulum-stand in any part. The hexagonal frame introduced into the box-frame is conspicuous in this view. The battery is not the same which was used at the mine. The mounting at the lower station was exactly similar, wanting only the journeyman-clock and the battery. The frame with agate-planes (represented on a larger scale), which is planted on the top of the box-frame, is supported on three screw-feet: the screw-stalks are perforated; two are cut with an internal screw-thread, and long screws are passed upwards through smooth holes in the box-frame, and act in these internal screw-threads and draw them firmly down: the perforation in the third is smooth, and the long screw which passes down through it acts in a screw-thread cut in the box-frame. At the sides of the blocks which carry the agate-planes are notched brass plates turning upon pins, connected by a stouter piece of brass beyond the pins, through which a screw passes that acts in the solid block below; by driving this screw the notches are raised, and engage with the ends of the pendulum-knife-edge, and lift it off the agates.

21. The system of observations which I proposed was the following. One of the invariable pendulums was to be mounted at the upper station and the other at the lower station, and the two pendulums were to be observed simultaneously by two observers. The "Swings" or series of vibrations were to follow each other incessantly, day and night, with no more interruption than would be required for observing the galvanic signals, reading thermometers, and making petty adjustments. Six Swings (each occupying in the gross four hours) were to be taken in each day. At concerted minutes of time before the first and after the last Swing, and between the end of each preceding Swing and the beginning of that which followed, galvanic signals were to be observed. Several "Coincidences" of the vibrations of the detached pendulum with the clock pendulum were to be observed in KATER's manner at the beginning and end of each Swing. This system was to be maintained during the whole efficient time of one week: then the pendulums with their agate-planes and

thermometers, but with no other apparatus, were to be interchanged, and a week's observations were to be made in this state. The pendulums were to be interchanged a second time and a third time, so that, in the whole, four series of observations would be taken: but I thought it probable that less than a week would suffice for each of these latter series.

22. On considering the amount of labour required for carrying out this plan, and considering also that it was not in my power to take an active part, I judged that six observers would be necessary. With the sanction of the Lords Commissioners of the Admiralty, I appointed Mr. EDWIN DUNKIN and Mr. WILLIAM ELLIS, Assistants of the Royal Observatory, as two of the observers. With the permission of M. J. JOHNSON, Esq., I was enabled to solicit and to obtain the services of Mr. NORMAN POGSON, Assistant of the Oxford Observatory; Professor CHALLIS consented to my asking the aid of Mr. G. S. CRISWICK, Assistant of the Cambridge Observatory; by permission of Professor CHEVALLIER, Mr. GEORGE RÜMKER, Astronomer of the Durham Observatory, joined the party; and finally I was enabled, by the kindness of R. C. CARRINGTON, Esq., to avail myself of the services of Mr. G. H. SIMMONDS, Assistant of the Red Hill Observatory. Mr. DUNKIN superintended the party, and, during the observations, controlled the local operations of every kind. Under the admirable management of Mr. DUNKIN, and the zealous and orderly assistance of all the gentlemen whom I have named, the work went on with the most perfect regularity. I cannot express how much I am indebted to the hearty cooperation of every individual Assistant during the whole course of the operations from the beginning to the end.

Lodgings were provided for the party in the town of South Shields. Their comfort, during their singular occupation, as well as my own on the occasions of my visits, were greatly increased by the uniform kindness of the authorities of the mine, and by the hospitable attentions of ROBERT INGHAM, Esq., M.P., JOHN ROBINSON, Esq., Mayor of South Shields, JAMES MATHER, Esq., J. C. STEVENSON, Esq., of the Jarrow Chemical Works, and other gentlemen resident in the town or neighbourhood. Attendants in the mine were selected and placed under the command of the observers by the owners of the mine.

23. I may now give a brief journal of the operations.

1854. August 5.—I examined the mine, and stations were selected.

Instruments were procured, repaired, and sent to the Harton Mine; and observers were collected.

September 26.—I went to South Shields.

September 27.—I erected the clocks and pendulum at the upper station.

September 28.—I erected the clocks and pendulum at the lower station. In the evening, four of the observers arrived.

September 29.—In the morning, the remaining observers arrived. All were employed in adjusting the pendulums, &c. and in the practice of observations.

1854. September 30.—I returned to Greenwich, leaving all in the charge of Mr. DUNKIN.

October 2.—Observations commenced in the morning with Swing 1. Pendulum 1821 above, No. 8 below. Observers for the day, Mr. DUNKIN (above), Mr. ELLIS (below), from the beginning of Swing 1 to the beginning of Swing 3.

Observers for the night, Mr. POGSON and Mr. RÜMKER, from the end of Swing 3 to the beginning of Swing 6.

October 3.—Observers for the day, Mr. CRISWICK and Mr. SIMMONDS, from the end of Swing 6 to the beginning of Swing 9: for the night, Mr. ELLIS and Mr. POGSON, from the end of Swing 9 to the beginning of Swing 12.

October 4.—Observers for the day, Mr. RÜMKER and Mr. CRISWICK, from the end of Swing 12 to the beginning of Swing 15: for the night, Mr. ELLIS and Mr. SIMMONDS, from the end of Swing 15 to the beginning of Swing 18.

October 5.—Observers for the day, Mr. CRISWICK and Mr. POGSON, from the end of Swing 18 to the beginning of Swing 21: for the night, Mr. SIMMONDS and Mr. RÜMKER, from the end of Swing 21 to the beginning of Swing 24.

October 6.—Observers for the day, Mr. DUNKIN and Mr. ELLIS, from the end of Swing 24 to the end of Swing 26. The First Series was closed at the end of Swing 26.

October 7.—Mr. DUNKIN, with the assistance of Mr. ELLIS and Mr. POGSON, interchanged the pendulums.

October 9.—The Second Series commenced with Swing 27. Pendulum No. 8 above, 1821 below.

Observers for the day, Mr. SIMMONDS and Mr. CRISWICK, from the beginning of Swing 27 to the beginning of Swing 29: for the night, Mr. ELLIS and Mr. DUNKIN, from the end of Swing 29 to the beginning of Swing 32.

October 10.—Observers for the day, Mr. RÜMKER and Mr. POGSON, from the end of Swing 32 to the beginning of Swing 35: for the night, Mr. CRISWICK and Mr. SIMMONDS, from the end of Swing 35 to the beginning of Swing 38.

October 11.—Observers for the day, Mr. DUNKIN and Mr. ELLIS, from the end of Swing 38 to the beginning of Swing 41: for the night, Mr. RÜMKER and Mr. POGSON, from the end of Swing 41 to the beginning of Swing 44.

October 12.—Observers for the day, Mr. SIMMONDS and Mr. CRISWICK, from the end of Swing 44 to the beginning of Swing 47: for the night, Mr. Dunkin and Mr. Ellis, from the end of Swing 47 to the beginning of Swing 50.

1854. October 13.—Observers for the day, Mr. Pogson and Mr. Rümker, from the end of Swing 50 to the end of Swing 52. The Second Series was closed at the end of Swing 52.

October 14.—Mr. Dunkin, with the assistance of Mr. Ellis and Mr. Pogson, interchanged the pendulums.

October 16.—The Third Series commenced with Swing 53. Pendulum 1821 above, No. 8 below. Observers for the day, Mr. Criswick and Mr. Simmonds, from the beginning of Swing 53 to the beginning of Swing 55: for the night, Mr. Rümker and Mr. Pogson, from the end of Swing 55 to the beginning of Swing 58.

October 17.—Observers for the day, Mr. Ellis and Mr. Dunkin, from the end of Swing 58 to the beginning of Swing 61: for the night, Mr. Simmonds and Mr. Criswick, from the end of Swing 61 to the beginning of Swing 64.

October 18.—Observers for the day, Mr. Pogson and Mr. Rümker, from the end of Swing 64 to the beginning of Swing 67; of which the end was observed by Mr. Dunkin and Mr. Ellis. The Third Series was closed at the end of Swing 67. Mr. Dunkin and Mr. Ellis interchanged the pendulums at night.

October 19.—The Fourth Series commenced with Swing 68, begun by Mr. Dunkin and Mr. Ellis. Pendulum No. 8 above, 1821 below. Observers for the day, Mr. Criswick and Mr. Simmonds, from the end of Swing 68 to the beginning of Swing 71: for the night, Mr. Rümker and Mr. Pogson, from the end of Swing 71 to the beginning of Swing 74.

October 20.—Observers for the day, Mr. Dunkin and Mr. Ellis, from the end of Swing 74 to the beginning of Swing 77: for the night, Mr. Simmonds and Mr. Criswick, from the end of Swing 77 to the beginning of Swing 80.

October 21.—Observers for the day, Mr. Pogson and Mr. Rümker, from the end of Swing 80 to the beginning of Swing 82: the end of Swing 82 was observed by Mr. Dunkin and Mr. Pogson. The Fourth Series and the whole operation closed with Swing 82.

October 22.—I arrived from Greenwich.

October 23.—I inspected the instruments in position, and dismounted them.

October 24.—The instruments were mounted in the Central Hall of South

Shields, and I explained the nature of the observations to an assembly of the residents of South Shields and its neighbourhood.

1854. October 25.—The instruments were finally dismounted, and packed up for return to Greenwich; and the party dispersed.

24. In terminating this general history of the operation, it is proper perhaps that I should briefly allude to the sources from which the expenses were defrayed.

The Royal Society, in granting the loan of their pendulums and clock, not only enabled me to undertake the operation with promptitude and with the security of using trustworthy instruments, but also removed one of the most serious causes of expense. I have already alluded to the extensive works constructed by the owners of the Harton Colliery, and which they made entirely at their own expense; and to the liberality of the Electric Telegraph Company, who gratuitously gave me their important assistance in the galvanic connexions. In transmitting the instruments from the wharfs at Newcastle to South Shields, Messrs. STEVENSON and Co., of the Jarrow Chemical Works, gave the use of their river-craft. The Mayor and Corporation of South Shields, unsolicited, caused the surveys to be made which were needed for computation of the attraction of the ground. Some parts of the apparatus (galvanometers, wires, stoves, &c.) were so evidently applicable to the prospective wants of the Royal Observatory, that they were not considered as a charge on the Experiment. The Lords Commissioners of the Admiralty, on my laying before them my proposals, contributed £100, which defrayed the greater part of the miscellaneous expenses. The residual charges were borne by myself.

SECTION II.—*Comparisons of the Upper and Lower Clocks.*

25. In the ordinary state, the galvanic circuit was interrupted at both circuit-breakers. At the pre-arranged minute for commencing signals, as nearly as possible, each of the observers completed the connexion, and then gave his attention to the galvanometer. The movement of the needle, at intervals of 15^s nearly, assured each observer that the other was prepared to observe; and every observation of the clock-time of the movement of the needle was then efficient for comparison of the two clocks. Only, it was necessary to know precisely, when the time of examining the written record of the time-signals should arrive, *which* were the *corresponding* observations of the same galvanic current. For this purpose, the following instructions were given. After observing the clock-times of a few signals, the upper observer interrupted the circuit during one signal only, and then restored the connexion. The lower observer, after waiting one or two minutes, did the same thing. After another short delay, the upper observer interrupted the circuit during two signals. The lower observer, after a short delay, did the same. Then the observations were continued till the upper observer was satisfied with the number, after which he definitively

broke the circuit; the lower observer then broke circuit; and both prepared to observe Coincidences of the pendulums. On examining the interruptions of signals, no difficulty was found in confronting the corresponding observations. In one instance only (between Swings 16 and 17) was the comparison totally lost, in consequence of one of the needles sticking fast. The observation of the time of the start of the needle is not very delicate; and this part of the operation would be made much more exact, by causing each clock to register its own seconds upon a common revolving barrel, in the manner of the American transits.

26. The observations on each side which had no corresponding observations on the other side being struck out, the means of all the times of the remaining observations in each group were taken, and the difference of means was formed. As a check, the differences of the times by the two clocks for each individual signal were formed, and the mean of the differences was taken. In this manner there were formed, for every group, mean corresponding times of SHELTON (the upper clock) and EARNSHAW (the lower clock); the differences of these, from group to group, gave the corresponding changes of clock-indications by the two clocks during the same period of absolute time; and the quotient $\frac{\text{EARNSHAW's change of indication}}{\text{SHELTON's change of indication}}$ gave the apparent ratio of rates. The logarithm of this ratio was formed, by means of CALLET's logarithms, to eight decimals.

27. A very cursory examination of these ratios showed that there was a considerable personal equation in the observation of the galvanometer-signals. Though (from the nature of the combination to be hereafter described) this scarcely produces an appreciable effect on the ultimate result, I thought it desirable to ascertain approximately its magnitude, and to apply the corresponding correction. I proceeded as follows:—In every instance in which the signal-observations at the beginning and at the end of a Swing were made by different observers, I compared the logarithms of apparent rates of $\frac{\text{EARNSHAW}}{\text{SHELTON}}$ during that swing with the logarithms for the preceding and following swings (in three instances, however, with only the preceding swing); and formed the excess of the logarithm of the intermediate swing above the mean of the preceding and following logarithms. This numerical excess may be compared with a symbolical formula, in which the symbols represent, not the actual error in time committed by each observer, but the logarithm (in 8-figure units) of the influence of his error on rates deduced from comparisons at 4-hour intervals. Twenty-seven equations were thus formed. By the method of minimum squares, these were reduced to six, of which (from the nature of the case) one was unnecessary, their sum being identically equal to zero. Assuming D (Mr. DUNKIN's error) $=0$, the others are found by solution of the equations. Though the equations are not very favourable, they suffice for giving very good corrections, and they show in particular that Mr. SIMMONDS recorded his times too early by nearly half a second. The following

Table exhibits this work ; the fundamental numbers being obtained from a table which is to follow :—

No. of Swing.	Order of Observers. Above.	Order of Observers. Below.	Symbol for the Errors of Logarithmic Rate $\frac{\text{EARNSHAW}}{\text{SHELTON}}$ produced by each observer's error in the estimation of time.	Errors inferred from the computed Logarithmic Rates.	No. of Swing.	Order of Observers. Above.	Order of Observers. Below.	Symbol for the Errors of Logarithmic Rate $\frac{\text{EARNSHAW}}{\text{SHELTON}}$ produced by each observer's error in the estimation of time.	Errors inferred from the computed Logarithmic Rates.
3.	D P	E R	D+R−P−E	− 277	47.	S D	C E	S+E−D−C	− 946
6.	P C	R S	P+S−C−R	− 45	50.	D P	E R	D+R−P−E	+ 135
9.	C E	S P	C+P−E−S	+ 134	55.	C R	S P	C+P−R−S	+2850
12.	E R	P C	E+C−R−P	−1957	58.	R E	P D	R+D−E−P	− 381
15.	R E	C S	R+S−E−C	+ 512	61.	E S	D C	E+C−S−D	− 254
18.	E C	S P	E+P−C−S	+2257	64.	S P	C R	S+R−P−C	−1621
21.	C S	P R	C+R−S−P	+1125	67.	P D	R E	P+E−D−R	+ 443
24.	S D	R E	S+E−D−R	+ 177	68.	D C	E S	D+S−C−E	−1079
29.	S E	C D	S+D−E−C	− 933	71.	C R	S P	C+P−R−S	+1156
32.	E R	D P	E+P−R−D	+ 360	74.	R D	P E	R+E−D−P	−1135
35.	R C	P S	R+S−C−P	−1581	77.	D S	E C	D+C−S−E	+1083
38.	C D	S E	C+E−D−S	+ 747	80.	S P	C R	S+R−P−C	−1706
41.	D R	E P	D+P−R−E	+ 956	82.	P D	R P	2P−D−R	− 465
44.	R S	P C	R+C−S−P	+ 531					

Equations.

$$
\begin{aligned}
18.C-2.D+2.E+P-4.R-15.S&=+10557\\
-2.C+15.D-14.E-5.P+5.R+S&=+577\\
+2.C-14.D+18.E+2.P-6.R-2.S&=-458\\
C-5.D+2.E+22.P-16.R-4.S&=+14048\\
-4.C+5.D-6.E-16.P+19.R+2.S&=-7873\\
-15.C+D-2.E-4.P+2.R+18.S&=-16851.
\end{aligned}
$$

Approximate Solution, assuming D=0

$$C=-\ 720$$
$$D=\ 0$$
$$E=-\ 240$$
$$P=+\ 180$$
$$R=-\ 330$$
$$S=-1500$$

These are applied, with signs changed, to correct the computed Logarithmic Rates in the Swings which are included in the last Table: and thus are formed the "Corrected Logarithmic Rates" in the Table which follows next.

28. It is impossible to give here in detail the whole of the observations of Galvanic Signals. The whole number of efficient signals observed at each station was 2454 (or 4908 observations of signals in all). It will be sufficient to give here the mean of each group (which, from the check described above, is extremely certain), and the calculations founded on those means.

It will be remarked that the pendulums of the clocks were altered at the times of interchanging the detached pendulums; that is, between Swings 26 and 27, between 52 and 53, and between 67 and 68.

No. of Swing.	Approximate Time (Astronomical Reckoning).	Number of Signals.	Mean of Times by SHELTON.	Mean of Times by EARNSHAW.	Interval by SHELTON.	Interval by EARNSHAW.	Rate $\frac{\text{EARNSHAW}}{\text{SHELTON}}$.	Logarithm of Rate $\frac{\text{EARNSHAW}}{\text{SHELTON}}$.	Corrected Logarithm of Rate $\frac{\text{EARNSHAW}}{\text{SHELTON}}$.
	Oct. h		h m s	h m s	h m s	h m s			
	1. 23	22	3 19 36·505	21 23 28·764					
1					4 0 23·100	4 0 38·722	1·0010831	0·00047012	
	2. 3	21	7 19 59·605	1 24 7·486					
2					3 58 21·652	3 58 37·400	1·0011011	0·00047793	
	2. 7	21	11 18 21·257	5 22 44·886					
3					4 45 27·829	4 45 46·421	1·0010855	0·00047117	0·00047387
	2. 11	29	16 3 49·086	10 8 31·307					
4					4 17 6·532	4 17 23·234	1·0010827	0·00046995	
	2. 16	17	20 20 55·618	14 25 54·541					
5					3 13 21·898	3 13 34·795	1·0011116	0·00048249	
	2. 19	25	23 34 17·516	17 39 29·336					
6					3 49 42·503	3 49 57·654	1·0010994	0·00047720	0·00047990
	2. 23	31	3 24 0·019	21 29 26·990					
7					3 55 2·071	3 55 17·433	1·0010893	0·00047282	
	3. 3	21	7 19 2·090	1 24 44·423					
8					4 2 41·510	4 2 57·445	1·0010944	0·00047503	
	3. 7	25	11 21 43·600	5 27 41·868					
9					4 31 5·786	4 31 23·591	1·0010947	0·00047516	0·00046316
	3. 11	22	15 52 49·386	9 59 5·459					
10					3 27 49·747	3 28 3·324	1·0010888	0·00047260	
	3. 15	24	19 20 39·133	13 27 8·783					
11					3 59 47·292	4 0 3·188	1·0011049	0·00047959	
	3. 19	24	23 20 26·425	17 27 11·971					
12					4 3 30·416	4 3 46·029	1·0010686	0·00046384	0·00047194
	3. 23	17	3 23 56·841	21 30 58·000					
13					3 58 21·005	3 58 37·058	1·0011225	0·00048723	
	4. 3	24	7 22 17·846	1 29 35·058					
14					5 14 58·735	5 15 19·267	1·0010865	0·00047160	
	4. 8	16	12 37 16·581	6 44 54·325					
15					2 40 24·234	2 40 34·859	1·0011040	0·00047920	0·00048790
	4. 11	13	15 17 40·815	9 25 29·184					
16 17	....	...	....	....	8 46 49·620	8 47 24·325	1·0010979	0·00047655	
	4. 20	31	0 4 30·435	18 12 53·509					
18					3 34 0·822	3 34 15·481	1·0011416	0·00049550	0·00047390
	4. 23	21	3 38 31·257	21 47 8·990					
19					3 44 11·953	3 44 26·496	1·0010812	0·00046930	
	5. 3	21	7 22 43·210	1 31 35·486					
20					3 58 15·444	3 58 31·268	1·0011071	0·00048054	
	5. 7	24	11 20 58·654	5 30 6·754					
21					4 3 26·760	4 3 43·156	1·0011225	0·00048723	0·00048453
	5. 11	29	15 24 25·414	9 33 49·910					
22					3 56 39·589	3 56 55·010	1·0010861	0·00047142	
	5. 15	30	19 21 5·003	13 30 44·920					
23					3 58 32·060	3 58 47·569	1·0010836	0·00047034	
	5. 19	19	23 19 37·063	17 29 32·489					
24					4 0 58·264	4 1 14·066	1·0010930	0·00047442	0·00048852
	5. 23	22	3 20 35·327	21 30 46·555					
25					3 59 49·427	4 0 5·170	1·0010942	0·00047495	
	6. 3	24	7 20 24·754	1 30 51·725					
26					4 0 15·008	4 0 30·854	1·0010993	0·00047716	
	6. 7	24	11 20 39·762	5 31 22·579					

TABLE (continued).

No. of Swing.	Approximate Time (Astronomical Reckoning).	Number of Signals.	Mean of Times by SHELTON.	Mean of Times by EARNSHAW.	Interval by SHELTON.	Interval by EARNSHAW.	Rate EARNSHAW/SHELTON.	Logarithm of Rate EARNSHAW/SHELTON.	Corrected Logarithm of Rate EARNSHAW/SHELTON.
	Oct. h		h m s	h m s	h m s	h m s			
27	8. 23	49	3 14 11·153	21 37 30·535	3 51 17·182	3 51 37·539	1·0014670	0·00063664	
28	9. 3	43	7 5 28·335	1 29 8·074	3 58 30·751	3 58 51·994	1·0014844	0·00064419	
29	9. 7	28	11 3 59·086	5 28 0·068	4 6 14·677	4 6 36·178	1·0014553	0·00063157	0·00063697
30	9. 11	24	15 10 13·763	9 34 36·246	4 0 41·885	4 1 3·098	1·0014690	0·00063751	
31	9. 15	25	19 10 55·648	13 35 39·344	3 52 14·148	3 52 34·667	1·0014726	0·00063907	
32	9. 19	27	23 3 9·796	17 28 14·011	4 18 25·029	4 18 47·889	1·0014744	0·00063985	0·00063715
33	9. 23	24	3 21 34·825	21 47 1·900	3 59 34·514	3 59 55·495	1·0014596	0·00063344	
34	10. 3	23	7 21 9·339	1 46 57·395	3 43 1·815	3 43 21·572	1·0014765	0·00064076	
35	10. 7	24	11 4 11·154	5 30 18·967	4 1 33·412	4 1 54·153	1·0014311	0·00062107	0·00063397
36	10. 11	39	15 5 44·566	9 32 13·120	3 58 38·771	3 58 59·655	1·0014586	0·00063300	
37	10. 15	32	19 4 23·337	13 31 12·775	3 59 55·890	4 0 16·912	1·0014603	0·00063374	
38	10. 19	39	23 4 19·227	17 31 29·687	4 15 19·657	4 15 42·268	1·0014760	0·00064054	0·00063514
39	10. 23	38	3 19 38·884	21 47 11·955	3 44 44·120	3 45 3·768	1·0014572	0·00063240	
40	11. 3	26	7 4 23·004	1 32 15·723	3 59 52·573	4 0 13·338	1·0014427	0·00062610	
41	11. 7	26	11 4 15·577	5 32 29·061	4 17 53·278	4 18 16·016	1·0014696	0·00063777	0·00063027
42	11. 11	22	15 22 8·855	9 50 45·077	3 46 55·016	3 47 14·790	1·0014524	0·00063031	
43	11. 15	24	19 9 3·871	13 37 59·867	3 55 13·879	3 55 34·324	1·0014486	0·00062866	
44	11. 19	22	23 4 17·750	17 33 34·191	4 15 16·791	4 15 39·072	1·0014547	0·00063131	0·00062861
45	11. 23	46	3 19 34·541	21 49 13·263	3 45 19·636	3 45 39·054	1·0014363	0·00062333	
46	12. 3	35	7 4 54·177	1 34 52·317	3 58 57·703	3 59 18·356	1·0014405	0·00062515	
47	12. 7	30	11 3 51·880	5 34 10·673	4 10 12·288	4 10 33·581	1·0014184	0·00061556	0·00062576
48	12. 11	28	15 14 4·168	9 44 44·254	3 53 8·084	3 53 28·225	1·0014399	0·00062489	
49	12. 15	29	19 7 12·252	13 38 12·479	3 57 45·889	3 58 6·269	1·0014286	0·00061999	
50	12. 19	27	23 4 58·141	17 36 18·748	4 22 20·477	4 22 42·963	1·0014286	0·00061999	0·00062269
51	12. 23	27	3 27 18·618	21 59 1·711	3 36 44·350	3 37 2·817	1·0014201	0·00061630	
52	13. 3	25	7 4 2·968	1 36 4·528	3 58 50·329	3 59 10·661	1·0014188	0·00061574	
	13. 7	29	11 2 53·297	5 35 15·189					
53	15. 23	104	2 51 48·661	22 3 1·492	3 32 42·619	3 32 54·360	1·0009200	0·00039937	
54	16. 3	25	6 24 31·280	1 35 55·852	4 0 3·781	4 0 16·936	1·0009134	0·00039650	
55	16. 7	26	10 24 35·061	5 36 12·788	4 2 14·546	4 2 28·746	1·0009770	0·00042410	0·00041120
56	16. 11	29	14 26 49·607	9 38 41·534	4 17 55·132	4 18 9·202	1·0009092	0·00039469	
57	16. 15	28	18 44 44·739	13 56 50·736	3 38 20·977	3 38 32·960	1·0009148	0·00039712	
58	16. 19	25	22 23 5·716	17 35 23·696	4 2 2·672	4 2 15·792	1·0009034	0·00039216	0·00039486
59	16. 23	25	2 25 8·388	21 37 39·488	3 58 57·599	3 59 10·638	1·0009095	0·00039482	
60	17. 3	23	6 24 5·987	1 36 50·126	4 0 55·113	4 1 8·229	1·0009074	0·00039390	
61	17. 7	29	10 25 1·100	5 37 58·355	4 1 48·703	4 2 1·816	1·0009039	0·00039238	0·00038698
62	17. 11	31	14 26 49·803	9 40 0·171	3 59 31·125	3 59 44·233	1·0009121	0·00039594	
63	17. 15	29	18 26 20·928	13 39 44·404	4 0 54·254	4 1 7·117	1·0008899	0·00038631	
64	17. 19	34	22 27 15·182	17 40 51·521	4 17 49·235	4 18 2·410	1·0008518	0·00036977	0·00038267
65	17. 23	29	2 45 4·417	21 58 53·931	3 42 13·263	3 42 25·106	1·0008884	0·00038565	
66	18. 3	30	6 27 17·680	1 41 19·037	3 58 48·764	3 59 1·081	1·0008597	0·00037320	
67	18. 7	27	10 26 6·444	5 40 20·118	4 3 7·643	4 3 20·333	1·0008699	0·00037763	0·00037493
	18. 11	45	14 29 14·087	9 43 40·451					
68	18. 19	55	21 34 41·845	17 32 42·435	4 1 58·198	4 2 16·440	1·0012565	0·00054535	0·00055075
69	18. 23	28	1 36 40·043	21 34 58·875	4 0 38·893	4 0 57·393	1·0012814	0·00055614	
70	19. 3	25	5 37 18·936	1 35 56·268	4 0 17·451	4 0 35·535	1·0012544	0·00054444	
71	19. 7	31	9 37 36·387	5 36 31·803	4 4 20·858	4 4 39·775	1·0012903	0·00056001	0·00054711
72	19. 11	56	13 41 57·245	9 41 11·578	3 54 17·299	3 54 35·192	1·0012729	0·00055246	
73	19. 15	27	17 36 14·544	13 35 46·770	4 1 31·831	4 1 50·051	1·0012573	0·00054570	
74	19. 19	28	21 37 46·375	17 37 36·821	4 11 36·610	4 11 55·090	1·0012241	0·00053129	0·00053879
75	19. 23	27	1 49 22·985	21 49 31·911	3 48 54·229	3 49 11·303	1·0012432	0·00053958	
76	20. 3	28	5 38 17·214	1 38 43·214	3 57 35·469	3 57 53·148	1·0012402	0·00053828	
77	20. 7	24	9 35 52·683	5 36 36·362	4 15 21·064	4 15 40·318	1·0012567	0·00054543	0·00053523
78	20. 11	30	13 51 13·747	9 52 16·680	3 47 1·059	3 47 17·720	1·0012232	0·00053091	
79	20. 15	31	17 38 14·806	13 39 34·400	3 58 48·581	3 59 6·177	1·0012281	0·00053302	
80	20. 19	30	21 37 3·387	17 38 40·577	4 5 56·270	4 6 13·720	1·0011826	0·00051330	0·00052620
81	20. 23	30	1 42 59·657	21 44 54·297	3 52 31·475	3 52 48·435	1·0012158	0·00052770	
82	21. 3	28	5 35 31·132	1 37 42·732	4 11 15·522	4 11 33·690	1·0012051	0·00052305	0·00051615
	21. 7	56	9 46 46·654	5 49 16·422					

SECTION III.—*General System of observing the Pendulums and of reducing the Observations.*

29. Before describing the observations, &c., I will remark that the pendulums above and below were mounted in exactly the same manner. Each angle of the iron stand, in each station, rested on a single brick; and great care was taken by Mr. DUNKIN that the bearing of these bricks should be perfectly solid. It was also a subject of Mr. DUNKIN's special attention to make the supplementary hexagonal iron frame (which I have described in article 15) quite firm; and, above all, to fix firmly at the beginning of each series and to examine carefully at the end of each series the frame carrying the agate-planes. In every instance these were found perfectly firm in their attachment. At every interchange of pendulums, Mr. DUNKIN carefully oiled and wiped the knife-edges and their agate-planes. At the beginning of every swing, the observer raised the knife-edge from contact with the agate-plane, by the screw-lifting apparatus, and then lowered it gently to a definite line of bearing.

30. The pendulum being observed in KATER's manner, by using the concealment of the bright disk on the clock-pendulum-bob behind the tail of the detached pendulum, when passing the aperture through which alone the disk can be seen, as indication of the Coincidence of the two pendulums in the times of passing their respective quiescent points; and supposing that there are trifling errors in the adjustments of position; it is seen in practice, or is shown by a very simple investigation, that the disk will first disappear in passing from one side (suppose the right side) towards the centre, then will disappear in passing from the left side (after which, if the errors of adjustment are in the proper direction, it will be invisible during several vibrations), then it will reappear on the left side, and will finally reappear on the right side. Either the mean of the times of the first and fourth phenomenon, or the mean of the second and third, or the mean of all four, may be used as the true time of Coincidence; and it was left to the discretion of the observers to adopt which they preferred. They chose, in every case, to observe the first and fourth only. This amounts in fact to using only one side of the aperture.

It is necessary for the success of this observation that the arc of the detached pendulum be less than that of the clock pendulum; in fact it was always much less. It is indifferent whether the detached pendulum vibrate quicker or slower than the clock pendulum; in fact it always vibrated slower.

31. Several Coincidences were always observed at the beginning of a Swing, and several at the end. These gave the Interval of Coincidences nearly enough to enable me to fix upon the number of intervening Coincidences. In general, some of the observed Coincidences were rejected, so that from two to five Coincidences were retained at the beginning, and a number at the end so corresponding that the difference of their means would represent an integral number of Intervals (thus there might be 3 at the beginning and 5 at the end, or 4 at the beginning and 2 at the

end). The mean of the first retained and the last retained being taken, the difference of means divided by the number of Intervals gave a very exact value of Mean Interval.

In the Interval, the detached pendulum lost two vibrations on the clock pendulum. If then the times of the first and last mean be taken *from the clock-face*, and if n be the number of seconds, referred to the clock, in the Mean Interval, the rate of the detached pendulum on the clock pendulum will be $\frac{n-2}{n}$. The following Tables contain all the values of log $\frac{n-2}{n}$ which are required here. For estimating the effect of any error in the Interval, it should be remembered that the logarithm of $\frac{86401}{86400}$, or of a rate of one second daily, is 500 in the last figures of 8-figure logarithms, nearly.

n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.
293·0	9·99702537	300·0	9·99709501	376·0	9·99768376	392·0	9·99777854	405·0	9·99785003
·1	2639	·1	9598	·1	8438	·1	7911	·1	5056
·2	2740	·2	9695	·2	8499	·2	7967	·2	5109
·3	2842	·3	9792	·3	8561	·3	8024	·3	5162
·4	2943	·4	9889	·4	8622	·4	8080	·4	5215
·5	3045	·5	9·99709986	·5	8684	·5	8137	·5	5268
·6	3146	·6	9·99710083	·6	8746	·6	8194	·6	5321
·7	3248	·7	0179	·7	8807	·7	8251	·7	5374
·8	3349	·8	0276	·8	8869	·8	8307	·8	5428
·9	3451	·9	0372	·9	8930	·9	8364	·9	5481
294·0	3552	301·0	0469	377·0	8992	393·0	8421	406·0	5534
·1	3653	·1	0565			·1	8477	·1	5587
·2	3754	·2	0662	389·0	9·99776137	·2	8534	·2	5639
·3	3856	·3	0758	·1	6194	·3	8590	·3	5692
·4	3957	·4	0855	·2	6252	·4	8647	·4	5744
·5	4058	·5	0951	·3	6309	·5	8703	·5	5797
·6	4158	·6	1047	·4	6367	·6	8759	·6	5850
·7	4259	·7	1143	·5	6424	·7	8816	·7	5903
·8	4359	·8	1239	·6	6482	·8	8872	·8	5955
·9	4460	·9	1335	·7	6539	·9	8929	·9	6008
295·0	4560	302·0	1431	·8	6597	394·0	8985	407·0	6061
				·9	6654			·1	6114
298·0	9·99707545	374·0	9·99767134	390·0	6712	403·0	9·99783932	·2	6166
·1	7643	·1	7196	·1	6769	·1	3986	·3	6219
·2	7741	·2	7258	·2	6827	·2	4040	·4	6271
·3	7840	·3	7321	·3	6884	·3	4094	·5	6324
·4	7938	·4	7383	·4	6942	·4	4148	·6	6377
·5	8036	·5	7445	·5	6999	·5	4202	·7	6429
·6	8134	·6	7507	·6	7056	·6	4255	·8	6482
·7	8232	·7	7569	·7	7113	·7	4308	·9	6534
·8	8330	·8	7632	·8	7170	·8	4362	408·0	6587
·9	8428	·9	7694	·9	7227	·9	4415	·1	6639
299·0	8526	375·0	7756	391·0	7284	404·0	4468	·2	6692
·1	8624	·1	7818	·1	7341	·1	4522	·3	6744
·2	8722	·2	7880	·2	7398	·2	4575	·4	6797
·3	8819	·3	7942	·3	7456	·3	4629	·5	6849
·4	8917	·4	8004	·4	7513	·4	4682	·6	6901
·5	9015	·5	8066	·5	7570	·5	4736	·7	6953
·6	9112	·6	8128	·6	7627	·6	4789	·8	7006
·7	9209	·7	8190	·7	7684	·7	4843	·9	7058
·8	9307	·8	8252	·8	7740	·8	4896	409·0	7110
·9	9404	·9	8314	·9	7797	·9	4950		

TABLE (continued).

n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.	n.	Log $\frac{n-2}{n}$.
488·0	9·99821645	489·3	9·99822120	490·6	9·99822591	491·9	9·99823062	493·2	9·99823529
·1	1682	·4	2156	·7	2628	492·0	3098	·3	3565
·2	1718	·5	2193	·8	2664	·1	3134	·4	3601
·3	1755	·6	2229	·9	2701	·2	3170	·5	3637
·4	1791	·7	2265	491·0	2737	·3	3206	·6	3673
·5	1828	·8	2302	·1	2773	·4	3242	·7	3708
·6	1864	·9	2338	·2	2809	·5	3278	·8	3744
·7	1901	490·0	2374	·3	2845	·6	3314	·9	3779
·8	1937	·1	2410	·4	2881	·7	3350	494·0	3815
·9	1974	·2	2446	·5	2917	·8	3385		
489·0	2010	·3	2483	·6	2953	·9	3421		
·1	2047	·4	2519	·7	2989	493·0	3457		
·2	2083	·5	2555	·8	3026	·1	3493		

32. The number given by this table is the logarithm of the mean Rate of the Detached Pendulum upon the Clock Pendulum under the actual circumstances of observation. It is next required to investigate the correction to this logarithm depending on the extent of the arc of vibration; one of the conditions of the data of the problem being, that the arc is observed only at the beginning and the end of the Swing.

In the first place, to compute the correction to the logarithm, supposing the arc of vibration constant. Let the whole arc of vibration, as seen upon the scale of inches, be I. For the pendulum 1821, suppose the scale to be placed 1 inch behind the pendulum; and for the pendulum 8, 1·8 inch behind the pendulum. And suppose the distance of the object-glass of the observing telescope to be 100 inches. Then the real whole arc of vibration is $\mathrm{I}\times\frac{100}{101}$ for pendulum 1821 and $\mathrm{I}\times\frac{100}{101{\cdot}8}$ for pendulum 8. The lengths of the two pendulums, from the knife-edge to the indicating point of the tail, are respectively 60·7 and 60·2 inches. Hence the proportion of the real whole arc of vibration to the length of the pendulum is $\mathrm{I}\times\frac{100}{101\times60{\cdot}7}$ for pendulum 1821 and $\mathrm{I}\times\frac{100}{101{\cdot}8\times60{\cdot}2}$ for pendulum 8; in which expressions the factors of I are sensibly the same. Call this proportion C. Then the number of vibrations observed is to be multiplied by $1+\frac{\mathrm{C}^2}{64}$ or $1+\mathrm{I}^2\times\frac{1}{64}\left(\frac{100}{101\times60{\cdot}7}\right)^2$. In the first instance therefore we require the logarithm of $1+\mathrm{I}^2\times\frac{1}{64}\left(\frac{100}{101\times60{\cdot}7}\right)^2$ for values of I not exceeding 2·5. The following Table contains the numbers required, in units of the last figures of 8-figure logarithms.

I.	Log.	I.	Log.	I.	Log.	I.	Log.	I.	Log.
0·1	2	0·6	65	1·1	218	1·6	462	2·1	796
0·2	7	0·7	88	1·2	260	1·7	522	2·2	874
0·3	16	0·8	116	1·3	305	1·8	585	2·3	955
0·4	29	0·9	146	1·4	353	1·9	652	2·4	1040
0·5	45	1·0	180	1·5	405	2·0	722	2·5	1128

Next, it is necessary to determine experimentally the law, or rather the numerical succession of values, in the diminution of the arc of vibration. For this purpose, observations were made at Greenwich on the extent of the whole arc in successive half-hours. The following Table contains (with sufficient approximation) the range of arc through each half-hour, the middle arc on which the correction may be supposed to depend, and the logarithmic correction as taken from the last table.

Range of Arc through half-hour.	Middle Arc.	Log. Correction.	Range of Arc through half-hour.	Middle Arc.	Log. Correction.	Range of Arc through half-hour.	Middle Arc.	Log. Correction.
2·35—1·97	2·14	831	0·71—0·59	0·64	81	0·22—0·19	0·20	7
1·97—1·64	1·79	579	0·59—0·49	0·54	52	0·19—0·16	0·17	6
1·64—1·38	1·50	402	0·49—0·41	0·45	36	0·16—0·13	0·14	4
1·38—1·16	1·26	287	0·41—0·34	0·37	26	0·13—0·10	0·11	3
1·16—0·98	1·06	205	0·34—0·29	0·31	18	0·10—0·08	0·09	2
0·98—0·83	0·90	146	0·29—0·25	0·27	13			
0·83—0·71	0·76	103	0·25—0·22	0·23	10			

Suppose now that we wished to find the logarithmic correction for a Swing whose first arc was 1·97 and whose last arc was 0·98. This Swing extends over four equal intervals of time, for which the log. corrections are respectively 579, 402, 287, 205. The log. correction therefore applicable to the whole time will be the mean of these four corrections, or 368. In a similar manner, we may obtain the correction with any other beginning and concluding arcs among those in the table above, and thus the next table is formed.

Logarithmic Correction for the whole Swing.

Concluding Arc.	Commencing Arc. 2·35	1·97	1·64	1·38	1·16	0·98	0·83	0·71	0·59	0·49	0·41	0·34	0·29	0·25	0·22	0·19	0·16	0·13	0·10
1·97	831																		
1·64	705	579																	
1·38	604	491	402																
1·16	525	423	345	287															
0·98	461	368	298	246	205														
0·83	408	324	260	213	176	146													
0·71	365	287	229	185	151	125	103												
0·59	329	258	204	164	134	110	92	81											
0·49	298	232	182	146	117	96	78	67	52										
0·41	272	210	164	130	104	84	63	56	44	36									
0·34	250	192	149	117	93	74	60	49	38	31	26								
0·29	231	176	136	106	83	66	53	43	33	27	22	18							
0·25	214	162	124	97	76	59	47	38	29	23	19	16	13						
0·22	199	151	115	89	69	54	42	34	26	21	17	14	12	10					
0·19	186	140	107	82	63	49	38	30	23	18	15	12	10	9	7				
0·16	175	131	99	76	59	45	35	28	21	17	13	11	9	8	6	6			
0·13	165	123	93	71	54	42	32	25	19	15	12	10	8	7	6	5	4		
0·10	156	116	87	66	51	38	30	23	17	14	11	9	7	6	5	4	4	3	
0·08	148	110	82	62	48	36	28	22	16	13	10	8	6	5	4	4	3	3	2

It was only necessary in fact to use so much of this table as is included between 2·35 and 1·38 for Commencing Arc, and between 0·98 and 0·34 for Concluding Arc. Between these limits, a skeleton table was prepared for every 0·01 in each argument, and was filled up by interpolation as far as was required, and no further. It is unnecessary to give it here, as all the essentials are contained in the table which has just been exhibited.

The application of this number to the logarithm of the mean Rate of the Detached Pendulum upon the Clock Pendulum, under the actual circumstances, gave the logarithm of mean Rate of the Detached Pendulum upon the Clock Pendulum, supposing the arc of vibration of the Detached Pendulum to have been indefinitely small.

The Commencing and Concluding Arcs which were used as Arguments for the table were those corresponding to the means of the Coincidences retained at the beginning and at the end of the Swing.

33. The next correction is that depending on the temperature of the Pendulum. On considering the slight discordance in the coefficients of expansion found by different experimenters, as well as the difficulty of exactly identifying the quality of the metal on which they experimented, it appeared to me best to adopt the result of Colonel SABINE (Experiments*, page 202—207), both because the method of experimenting was precisely the same as the method of using the pendulum in these operations, and because there can scarcely be a doubt that the metal was similar, as nearly as is possible in different bars. A small correction is required to Colonel SABINE's results, because at the time of his drawing the conclusion as to the effect of temperature, the ancient erroneous computation for the effect of buoyancy was still in use. Adopting his multiplier 1·655 of the correction computed for mere statical buoyancy, as applicable to pendulums of the same form as those used in these experiments †, the corrections to the numbers in the "Experiments" are as follows:—

Page 202, *for* 6·25 *read* 10·34; which gives for Pendulum 3, 86166·49.
Page 203, *for* 6·2 *read* 10·26; which gives for Pendulum 4, 86174·99.
Page 205, *for* 5·65 *read* 9·35; which gives for Pendulum 3, 86149·61.
Page 206, *for* 5·7 *read* 9·43; which gives for Pendulum 4, 86159·35.
Page 207, the results for 1° FAHRENHEIT will be respectively 0·4318 and 0·4300. The mean 0·4309 corresponds to 86166 vibrations in one day.

Hence, to reduce the vibrations observed at the temperature t° of FAHRENHEIT to the vibrations which would have been observed in air of the same density at temperature 50°, the number of observed vibrations must be multiplied by $\frac{86166+(t-50)\times 0{\cdot}4309}{86166}$, or by $1+(t-50)\times 0{\cdot}00000501$. I do not at present form the logarithm of this quantity, as there will be another term depending on temperature, introduced by the consideration of the buoyancy-correction, which will be combined with this.

* "An Account of Experiments to determine the Figure of the Earth by means of the Pendulum vibrating seconds in different Latitudes, as well as on various other subjects of Philosophical Inquiry. By EDWARD SABINE, &c. London, 1825."

† Philosophical Transactions, 1829.

34. The next correction is that for the density of the atmosphere. If we adopt Sir GEORGE SHUCKBURGH's elements (which are abundantly accurate for this purpose), the barometer-reading being B (expressed in English inches) at the temperature t° of FAHRENHEIT, its reading at temperature 53° would be

$$\frac{B}{29{\cdot}27}(29{\cdot}27-\overline{t-53}\times 0{\cdot}002615)=B(1-\overline{t-53}\times 0{\cdot}0000896).$$

The proportion of the weight of air in this state to that of air at barometer-reading $29^{\text{in.}}{\cdot}27$, thermometer 53° (Sir GEORGE SHUCKBURGH's standard elements), will be

$$\frac{B(1-\overline{t-53}\times 0{\cdot}0000896)}{29{\cdot}27}\times\left(1+\frac{53-t}{480}\right)=\frac{B}{29{\cdot}27}(1-\overline{t-53}\times 0{\cdot}002173)$$
$$=\frac{B}{29{\cdot}27}(1{\cdot}006519-\overline{t-50}\times 0{\cdot}002173).$$

With the elements $29^{\text{in.}}{\cdot}27$ and 53°, the weight of air is $\frac{1}{836}$ that of water; and, with KATER's specific gravity 8·469, the weight of air is $\frac{1}{836}\times\frac{1}{8{\cdot}469}$ that of the pendulum; the effect of this on the vibrations of the pendulum, adopting Colonel SABINE's factor 1·655 of the statical buoyancy, will be to diminish them by the part

$$\frac{B\times 1{\cdot}655}{1672\times 8{\cdot}469\times 29{\cdot}27}\times(1{\cdot}006519-\overline{t-50}\times 0{\cdot}002173).$$

In the small term multiplied by $t-50$, we may consider $\frac{B}{29{\cdot}27}=\frac{26}{25}$. Then the diminution of the number of vibrations will be

$$\frac{B\times 1{\cdot}655\times 1{\cdot}006519}{1672\times 8{\cdot}469\times 29{\cdot}27}-\overline{t-50}\cdot\frac{0{\cdot}00226\times 1{\cdot}655}{1672\times 8{\cdot}469},\text{ or } B\times 0{\cdot}000004019-\overline{t-50}\times 0{\cdot}00000026.$$

In order to correct the number of vibrations observed, so as to produce the number of vibrations which would take place in vacuum, we must multiply the number observed by

$$1+B\times 0{\cdot}000004019-\overline{t-50}\times 0{\cdot}00000026.$$

35. Combining this factor with the factor depending on the temperature of the pendulum, or $1+\overline{t-50}\times 0{\cdot}00000501$, the complete factor is

$$1+B\times 0{\cdot}000004019+\overline{t-50}\times 0{\cdot}00000475,$$

or

$$(1+B\times 0{\cdot}000004019)\times(1+\overline{t-50}\times 0{\cdot}00000475).$$

The logarithms of these factors are $B\times 0{\cdot}0000017453$ and $\overline{t-50}\times 0{\cdot}000002063$. The following Tables will suffice for the examination of the corrections in the succeeding Section.

Thermometer.	Correction to Log. Rate.	Thermometer.	Correction to Log. Rate.
50°·0	0·00000000	60°·0	0·00002063
·5	103	·5	2166
51·0	207	61·0	2269
·5	310	·5	2372
52·0	413	62·0	2475
·5	516	·5	2578
53·0	619	63·0	2681
·5	722	·5	2784
54·0	825	64·0	2887
·5	928	·5	2990
55·0	1032	65·0	3094
·5	1135	·5	3197
56·0	1238	66·0	3300
·5	1341	·5	3403
57·0	1444	67·0	3506
·5	1547	·5	3609
58·0	1650	68·0	3712
·5	1753	·5	3815
59·0	1856	69·0	3918
·5	1960	·5	4022
60·0	2063	70·0	4125

Barometer.	Correction to Log. Rate.
in. 29·0	0·00005061
·2	5096
·4	5131
·6	5166
·8	5201
30·0	5236
·2	5271
·4	5306
·6	5340
·8	5375
31·0	5410
·2	5445
·4	5480
·6	5514
·8	5549
32·0	5584

The "Corrected Log. Rate of Pendulum upon Clock," or Log. Rate supposing that the temperature of the Pendulum is 50°, and that it vibrates in vacuum, in an indefinitely small arc, is found by adding together the logarithm of $\frac{n-2}{2}$, the correction for arc, the correction for thermometer, and the correction for barometer, all taken from the tables above.

Section IV.—*Abstract of the Pendulum Observations at the Upper Station.*

36. The remarks in the preceding Sections explain all the essential points in the following Table. It is only necessary to add, that the temperature of the Barometer (which was suspended in the Anteroom, very near to the Stove) was higher than that of the Pendulum; and a small correction, never exceeding −0·075, has been applied to its reading.

First Series. Pendulum 1821 above.

No. of Swing.	Number of Coincidences.		Number of Intervals.	Mean Interval.	Arc of Vibration.		Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on SHELTON.
	Beginning.	End.			Beginning.	End.			
				s			°	in.	
1	3	3	22	493·182	1·91	0·58	57·81	29·930	9·99830603
2	3	3	21	492·341	1·75	0·60	58·78	29·856	466
3	3	3	28	491·923	1·90	0·44	58·81	29·729	284
4	3	2	23	491·399	1·85	0·59	59·38	29·537	209
5	3	2	17	491·711	1·79	0·67	58·84	29·503	211
6	3	3	22	492·470	1·88	0·63	58·30	29·658	409
7	3	3	22	492·553	1·71	0·59	58·26	29·763	410
8	3	3	22	492·932	1·64	0·50	57·54	29·834	379
9	2	2	27	493·352	1·77	0·42	56·88	29·896	406
10	3	3	20	493·450	1·83	0·66	56·79	29·895	488
11	3	3	24	493·167	1·98	0·55	57·64	29·861	556
12	3	2	24	492·719	2·13	0·60	57·00	29·813	295
13	3	3	22	493·235	1·92	0·61	56·98	29·700	421
14	3	3	23	492·391	2·09	0·55	57·44	29·553	201
15	3	3	20	492·759	1·88	0·62	57·89	29·488	394
16	3	3	22	492·507	1·99	0·61	58·25	29·395	379
17	3	3	22	492·371	1·88	0·59	58·61	29·298	363
18	3	3	21	492·126	1·91	0·71	58·55	29·218	282
19	3	3	20	492·625	1·88	0·65	58·78	29·243	495
20	3	3	23	493·189	1·85	0·58	58·10	29·272	541
21	3	3	23	493·217	1·87	0·63	57·94	29·318	542
22	4	3	20	493·775	1·70	0·57	57·55	29·366	628
23	3	3	21	493·722	1·91	0·58	57·13	29·438	569
24	3	3	24	493·882	2·02	0·56	56·55	29·537	539
25	3	3	22	493·879	1·95	0·63	56·41	29·622	529
26	3	3	24	493·764	2·03	0·56	56·44	29·702	504

Second Series. Pendulum 8 above.

No. of Swing.	Coincidences Beginning.	Coincidences End.	Number of Intervals.	Mean Interval.	Arc Beginning.	Arc End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on SHELTON.
27	4	4	27	376·519	1·96	0·60	54·18	29·699	9·99775000
28	4	4	29	376·082	1·97	0·66	55·71	29·725	5069
29	4	4	33	375·803	2·03	0·54	56·13	29·770	4974
30	4	4	28	375·647	1·85	0·62	56·58	29·820	4965
31	3	3	31	375·650	2·00	0·62	56·53	29·864	4990
32	4	4	33	375·534	2·03	0·52	56·45	29·892	4887
33	3	3	29	375·511	1·93	0·61	58·05	29·916	5211
34	3	2	31	375·191	2·00	0·63	59·26	29·908	5279
35	3	3	32	375·162	1·77	0·51	59·84	29·869	5306
36	4	4	28	374·938	1·82	0·61	60·01	29·808	5223
37	4	4	29	375·172	2·00	0·62	59·48	29·878	5303
38	4	4	33	375·424	1·90	0·64	57·95	30·024	5157
39	4	4	29	375·397	1·98	0·68	57·68	30·134	5127
40	4	4	31	375·665	1·93	0·60	57·76	30·235	5299
41	3	2	35	375·807	2·03	0·54	57·04	30·358	5262
42	3	3	30	376·428	1·90	0·51	56·43	30·445	5505
43	4	4	30	376·692	1·81	0·54	55·36	30·518	5454
44	4	4	32	376·902	1·95	0·53	55·23	30·567	5583
45	3	3	27	376·729	2·07	0·73	55·30	30·561	5566
46	4	4	30	376·458	1·98	0·63	55·84	30·557	5466
47	4	4	34	376·507	1·99	0·57	55·95	30·535	5502
48	4	4	30	376·425	1·84	0·57	56·56	30·493	5545
49	4	4	30	376·308	1·96	0·62	56·61	30·430	5504
50	4	4	34	376·158	2·03	0·55	56·78	30·399	5437
51	4	4	28	376·304	1·88	0·64	56·86	30·368	5533
52	2	3	33	376·126	1·91	0·52	57·38	30·343	5503

Third Series. Pendulum 1821 above.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Interval.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on SHELTON.
				s			°	in.	
53	3	3	19	489·000	1·99	0·71	53·05	29·916	9·99828152
54	3	3	23	488·775	2·00	0·60	53·30	29·811	8078
55	3	3	24	488·805	1·91	0·53	53·59	29·721	8100
56	3	3	26	489·397	1·76	0·46	53·51	29·638	8244
57	3	3	21	489·222	1·91	0·62	53·39	29·556	8204
58	3	3	25	489·480	2·00	0·53	52·84	29·542	8175
59	3	3	22	489·311	1·81	0·60	52·75	29·529	8080
60	3	3	23	488·848	1·98	0·59	53·18	29·499	8020
61	3	3	23	488·717	2·05	0·65	52·99	29·454	7954
62	3	3	22	489·038	1·72	0·60	53·26	29·493	8063
63	3	3	22	489·303	1·94	0·59	53·10	29·544	8171
64	3	3	27	489·568	2·00	0·48	52·75	29·599	8186
65	3	3	22	489·409	1·97	0·67	52·73	29·633	8172
66	3	3	25	489·553	2·03	0·58	52·59	29·649	8188
67	3	3	25	489·826	1·81	0·54	51·85	29·662	8090
				Fourth Series. Pendulum 8 above.					
68	5	5	29	375·879	2·00	0·61	51·53	29·662	9·99774063
69	5	5	29	376·117	1·81	0·58	51·25	29·608	4102
70	5	5	29	375·855	1·80	0·60	51·65	29·481	4004
71	5	5	29	375·517	1·88	0·63	52·11	29·333	3885
72	5	5	28	375·804	1·74	0·57	53·28	29·279	4258
73	5	5	29	375·700	1·71	0·56	53·28	29·250	4179
74	3	3	34	375·480	1·87	0·53	53·03	29·245	4010
75	5	5	26	375·208	1·75	0·68	53·43	29·250	3938
76	5	5	28	374·893	2·02	0·74	53·46	29·294	3824
77	4	4	32	374·953	1·91	0·59	53·39	29·381	3802
78	5	5	26	375·273	1·90	0·74	52·95	29·447	3956
79	5	5	28	375·332	1·98	0·72	52·65	29·502	3950
80	4	4	33	375·493	1·91	0·50	52·56	29·561	3976
81	5	5	28	375·236	1·92	0·68	53·66	29·559	4089
82	5	5	30	375·213	1·85	0·59	54·08	29·538	4122

SECTION V.—*Abstract of the Pendulum Observations at the Lower Station.*

37. No correction was applied to the immediate readings of the instruments, except that the Barometer-reading was increased by 0in.·016, which was required to make its indications correspond to those of the upper Barometer.

First Series. Pendulum 8 below.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Intervals.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on EARNSHAW.
				s			°	in.	
1	3	5	31	293·542	2·01	0·73	63·55	31·356	9·99711658
2	5	5	34	293·600	2·03	0·67	63·71	31·261	718
3	4	2	64	293·891	1·97	0·36	63·70	31·155	902
4	4	5	24	293·369	1·96	0·97	63·61	30·900	475
5	5	4	30	293·434	1·78	0·67	63·60	30·897	420
6	4	4	35	293·314	2·00	0·68	63·90	31·079	433
7	4	4	35	293·375	1·55	0·51	63·92	31·205	409
8	4	4	36	293·125	1·93	0·84	64·16	31·274	361
9	4	3	46	293·266	1·92	0·49	63·94	31·337	374
10	5	3	33	293·276	1·77	0·73	63·55	31·372	344
11	4	4	40	293·281	1·74	0·53	63·50	31·327	280
12	3	3	39	293·385	1·82	0·58	63·43	31·234	379
13	5	5	36	293·261	1·76	0·61	63·44	31·068	222
14	5	5	38	293·253	1·82	0·59	63·33	30·940	176
15	5	3	39	293·275	1·85	0·63	63·45	30·880	227
16	4	4	37	293·216	2·00	0·73	63·44	30·807	202
17	4	4	39	293·250	1·98	0·69	63·46	30·701	209
18	4	4	36	293·337	1·96	0·72	63·39	30·628	273
19	3	3	39	293·350	1·93	0·66	63·49	30·627	288
20	4	4	39	293·324	1·78	0·58	63·43	30·663	211
21	4	4	38	293·349	1·90	0·52	63·05	30·713	172
22	5	3	38	293·283	1·89	0·63	63·31	30·752	191
23	5	5	37	293·376	1·90	0·60	63·48	30·824	328
24	5	3	42	293·502	1·83	0·49	63·64	30·938	470
25	4	4	39	293·474	2·00	0·60	63·20	31·022	419
26	5	5	40	293·425	1·93	0·61	63·16	31·132	370
Second Series. Pendulum 1821 below.									
27	4	4	26	389·510	1·95	0·69	62·91	31·118	9·99784802
28	4	4	29	389·452	2·15	0·65	63·14	31·139	4849
29	3	3	32	389·370	2·17	0·65	63·44	31·212	4879
30	4	4	27	389·509	1·95	0·70	63·25	31·260	4898
31	4	4	27	389·644	1·94	0·74	63·06	31·283	4951
32	4	4	32	389·992	1·80	0·51	63·13	31·313	5091
33	4	4	31	390·012	1·97	0·59	63·31	31·310	5186
34	4	4	28	390·210	1·87	0·64	63·31	31·288	5291
35	4	4	30	390·333	1·88	0·56	63·51	31·255	5380
36	4	4	29	390·314	1·99	0·67	63·66	31·195	5434
37	4	4	28	390·602	1·81	0·59	63·64	31·261	5557
38	3	3	33	390·692	2·01	0·51	63·60	31·454	5647
39	3	3	29	390·885	1·97	0·65	63·53	31·571	5790
40	3	3	32	391·172	1·82	0·55	63·50	31·694	5921
41	4	4	34	391·397	1·91	0·54	63·48	31·799	6076
42	4	4	26	391·615	1·76	0·69	63·31	31·902	6194
43	4	4	30	391·800	1·84	0·63	63·28	31·966	6302
44	3	3	34	392·069	1·99	0·56	63·16	32·019	6449
45	4	4	25	392·125	2·01	0·76	63·54	32·014	6615
46	4	4	29	392·422	1·88	0·59	63·30	31·992	6660
47	3	3	31	392·554	2·07	0·64	63·29	31·983	6778
48	4	4	28	392·705	1·96	0·75	62·84	31·943	6774
49	4	4	29	392·927	2·03	0·75	62·70	31·894	6877
50	4	3	33	393·396	1·77	0·48	63·15	31·831	7109
51	4	4	26	393·755	1·71	0·59	63·63	31·794	7421
52	4	4	29	394·013	1·64	0·59	63·43	31·763	7509

Third Series. Pendulum 8 below.

No. of Swing.	Number of Coincidences. Beginning.	End.	Number of Intervals.	Mean Intervals.	Arc of Vibration. Beginning.	End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on EARNSHAW.
				s			°	in.	
53	5	5	28	298·521	1·96	0·90	62·05	31·335	9·99716353
54	5	5	37	298·819	1·89	0·67	61·85	31·229	6506
55	4	4	42	298·929	2·05	0·63	61·84	31·135	6615
56	4	4	45	299·144	1·92	0·54	61·68	31·049	6732
57	4	5	34	299·342	1·83	0·68	61·61	30·987	6919
58	3	3	40	299·479	2·05	0·67	61·81	30·971	7129
59	5	5	38	299·661	1·89	0·66	61·75	30·953	7259
60	5	5	38	299·832	1·98	0·67	61·65	30·922	7416
61	4	4	40	299·950	2·08	0·65	61·71	30·883	7553
62	5	5	36	300·150	1·92	0·65	62·04	30·907	7788
63	5	5	36	300·300	1·89	0·65	62·08	30·970	7946
64	4	4	43	300·468	1·97	0·59	62·30	31·021	8162
65	5	5	35	300·723	1·84	0·68	62·53	31·050	8472
66	5	5	37	301·449	1·91	0·67	62·80	31·084	9234
67	3	3	42	301·730	2·11	0·61	62·64	31·108	9497

Fourth Series. Pendulum 1821 below.

No. of Swing.	Number of Coincidences. Beginning.	End.	Number of Intervals.	Mean Intervals.	Arc of Vibration. Beginning.	End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rates of Pendulum on EARNSHAW.
68	4	4	29	403·647	1·98	0·63	61·55	31·119	9·99792364
69	5	5	25	403·892	2·08	0·79	61·73	31·045	2585
70	4	4	27	404·338	1·89	0·59	61·81	30·893	2722
71	3	3	30	404·817	1·95	0·57	61·64	30·746	2922
72	5	5	25	405·256	1·68	0·63	61·66	30·680	3117
73	5	5	27	405·578	1·79	0·64	61·65	30·652	3300
74	4	4	31	405·984	1·80	0·52	61·75	30·651	3512
75	4	2	28	406·281	1·81	0·58	61·88	30·661	3712
76	4	4	29	406·677	1·81	0·54	61·96	30·703	3937
77	3	3	32	407·141	1·91	0·49	62·00	30·796	4208
78	5	5	25	407·372	1·85	0·67	62·19	30·868	4414
79	5	5	26	407·835	1·80	0·60	62·20	30·916	4645
80	3	3	30	408·133	2·13	0·64	62·28	30·982	4897
81	5	5	27	408·511	1·91	0·64	62·73	30·969	5146
82	4	4	31	409·032	2·07	0·59	62·31	30·909	5338

SECTION VI.—*Computation of Logarithmic Rate of Lower Pendulum upon Upper Pendulum; combination of individual results; and conclusion on the proportion of gravity at the Lower Station to gravity at the Upper Station.*

38. The quantity now to be found is $\text{Log.}\frac{\text{Rate of Lower Pendulum}}{\text{Rate of Upper Pendulum}}$. This is

$$=\text{Log.}\frac{\text{Rate of Lower Pendulum}}{\text{Rate of EARNSHAW}}+\text{Log.}\frac{\text{Rate of EARNSHAW}}{\text{Rate of SHELTON}}-\text{Log.}\frac{\text{Rate of Upper Pendulum}}{\text{Rate of SHELTON}}.$$

The first of these quantities is given, for every Swing, in Section V.; the second in Section II.; and the third in Section IV. Thus the following numbers are formed.

First Series. Log. Rate of Pendulum 8 below on Pendulum 1821 above.

No. of Swing.	Log. Rate $\frac{\text{Lower Pendulum}}{\text{Upper Pendulum}}$.	No. of Swing.	Log. Rate $\frac{\text{Lower Pendulum}}{\text{Upper Pendulum}}$.	No. of Swing.	Log. Rate $\frac{\text{Lower Pendulum}}{\text{Upper Pendulum}}$.	No. of Swing.	Log. Rate $\frac{\text{Lower Pendulum}}{\text{Upper Pendulum}}$.
1	9·99928067	8	9·99928485	15	9·99929623	22	9·99927705
2	9045	9	7284	16	8478	23	7793
3	9005	10	8116	17	8501	24	9783
4	8261	11	8683	18	8381	25	8385
5	9458	12	8278	19	7723	26	8582
6	9014	13	9524	20	8724		
7	8281	14	8135	21	9083		

Second Series. Log. Rate of Pendulum 1821 below on Pendulum 8 above.

No. of Swing.	Log. Rate	No. of Swing.	Log. Rate	No. of Swing.	Log. Rate	No. of Swing.	Log. Rate
27	0·00073466	34	0·00074088	41	0·00073841	48	0·00073718
28	4199	35	3471	42	3720	49	3371
29	3602	36	3511	43	3714	50	3941
30	3684	37	3628	44	3727	51	3518
31	3868	38	4004	45	3382	52	3580
32	3919	39	3903	46	3709		
33	3319	40	3232	47	3852		

Third Series. Log. Rate of Pendulum 8 below on Pendulum 1821 above.

No. of Swing.	Log. Rate	No. of Swing.	Log. Rate	No. of Swing.	Log. Rate	No. of Swing.	Log. Rate
53	9·99928138	57	9·99928427	61	9·99928297	65	9·99928865
54	8078	58	8440	62	9319	66	8366
55	9635	59	8661	63	8406	67	8900
56	7957	60	8786	64	8243		

Fourth Series. Log. Rate of Pendulum 1821 below on Pendulum 8 above.

No. of Swing.	Log. Rate	No. of Swing.	Log. Rate	No. of Swing.	Log. Rate	No. of Swing.	Log. Rate
68	0·00073376	72	0·00074105	76	0·00073941	80	0·00073541
69	4097	73	3691	77	3929	81	3827
70	3162	74	3381	78	3549	82	2831
71	3748	75	3732	79	3997		

39. On tracing the irregularities in these numbers to their sources, it will be seen that they arise almost entirely from the irregularities in the comparisons of the clocks. The rate of each pendulum upon its clock is either so constant, or changes by such uniform degrees in the same direction, that there is every reason to presume on the extreme steadiness both of the detached pendulums and of the clocks.

Remarking then that, when a large variable error is combined with a small variable error, the magnitude of the probable error in the combination is scarcely affected by the small error, we may treat these irregularities of result as if they were entirely due to irregularities of comparison; and we have now to investigate the rule to be followed in combining the special results, supposed to be erroneous from that cause only, in order to obtain a final result whose probable error shall be the smallest possible. It is evident that we are not to give equal weights to the different results, for we should thus neglect all comparisons except the first and the last.

40. Let the comparisons be numbered

$$0, \quad 1, \quad 2, \quad 3, \quad 4, \quad \ldots\ldots\ldots\ldots \overline{n-1}, \quad n,$$

and let the swings be numbered

$$1, \quad 2, \quad 3, \quad 4, \quad \ldots\ldots\ldots\ldots n.$$

Let the *actual* errors of comparison, estimated by their effect on a four-hours' rate, in units of the last figure of 8-figure logarithms, be

$$E_0, \quad E_1, \quad E_2, \quad E_3, \quad E_4, \quad \ldots\ldots\ldots\ldots E_{n-1}, \quad E_n,$$

(where E_{16} is supposed to be taken as if the comparison, which was really omitted, agreed with that produced by interpolation between the two neighbouring comparisons,) and let the *probable* errors of comparison be

$$e_0, \quad e_1, \quad e_2, \quad e_3, \quad e_4, \quad \ldots\ldots\ldots\ldots e_{n-1}, \quad e_n,$$

and let the weights for the results of the separate Swings be

$$w_1, \quad w_2, \quad w_3, \quad w_4, \quad \ldots\ldots\ldots\ldots w_n.$$

Then the errors in the results of the separate Swings, produced by the errors of comparisons, are

$$(E_1-E_0), \quad (E_2-E_1), \quad (E_3-E_2), \quad \ldots\ldots\ldots\ldots (E_n-E_{n-1});$$

and, combining these with the weights

$$w_1, \quad w_2, \quad w_3, \quad \ldots\ldots\ldots\ldots w_n,$$

the *actual* error of the final result will be

$$\frac{w_1(E_1-E_0)+w_2(E_2-E_1)+w_3(E_3-E_2)+\ldots\ldots\ldots+w_n(E_n-E_{n-1})}{w_1 \quad +w_2 \quad +w_3 \quad +\ldots\ldots\ldots\ldots+w_n}$$

$$=\frac{-w_1E_0+(w_1-w_2)E_1+(w_2-w_3)E_2+(w_3-w_4)E_3\ldots\ldots\ldots+w_nE_n}{w_1 \quad +w_2 \quad +w_3 \quad +\ldots\ldots\ldots\ldots+w_n}.$$

Hence, by the well-known rules of the Calculus of Probabilities, the square of the *probable* error of the final result will be

$$\frac{w_1^2.e_0^2+(w_1-w_2)^2.e_1^2+(w_2-w_3)^2e_2^2+(w_3-w_4)^2e_3^2+\ldots\ldots\ldots+w_n^2.e_n^2}{(w_1 \quad + \quad w_2 \quad + \quad w_3 \quad +\ldots\ldots\ldots\ldots+w_n)^2}.$$

And if, for simplicity, we suppose all the comparisons equally good, so that for e_0, e_1, e_2, &c. we may put e, the expression for the square of the *probable* error of the final result becomes

$$e^2\times\frac{w_1^2+(w_1-w_2)^2+(w_2-w_3)^2+\ldots\ldots\ldots+w_n^2}{(w_1 \quad + \quad w_2 \quad +.w_3 \quad +\ldots\ldots+w_n)^2}:$$

and we have now to determine values of w_1, w_2, &c., which will make this square of the final probable error a minimum.

For convenience, call this fraction $e^2 \times \frac{N}{S^2}$.

Differentiate with respect to w_1, and make the differential coefficient $=0$;

$$S \times (2w_1 - w_2) - N = 0, \text{ or } 2w_1 - w_2 = \frac{N}{S}.$$

Differentiate with respect to w_2, and make the differential coefficient $=0$;

$$S \times (2w_2 - w_1 - w_3) - N = 0, \text{ or } 2w_2 - w_1 - w_3 = \frac{N}{S}$$

$$\text{Similarly } 2w_3 - w_2 - w_4 = \frac{N}{S}$$

$$\cdots\cdots\cdots$$

$$2w_{n-1} - w_{n-2} - w_n = \frac{N}{S}$$

$$2w_n - w_{n-1} = \frac{N}{S}.$$

Let $\frac{N}{S} = 2b$, the value of b being at present unknown or perhaps arbitrary. Then

$$\begin{aligned} w_1 &= w_1 &&= w_1 \\ w_2 &= 2w_1 - 2b &&= 2w_1 - 2b \\ w_3 &= 2w_2 - w_1 - 2b &&= 3w_1 - 6b \\ w_4 &= 2w_3 - w_2 - 2b &&= 4w_1 - 12b \\ &\cdots\cdots\cdots \\ w_{n-1} &= && (n-1)w_1 - (n-1)(n-2)b \\ w_n &= && nw_1 - n(n-1)b. \end{aligned}$$

Substituting the two last in the equation $2w_n - w_{n-1} = 2b$, we obtain

$$w_1 = nb,$$

and, substituting this in the other expressions,

$$\begin{aligned} w_2 &= (2n - 2)b \\ w_3 &= (3n - 6)b \\ w_4 &= (4n - 12)b \\ w_5 &= (5n - 20)b \\ &\text{\&c.} \end{aligned}$$

of which the law is evident. The second difference is constant, and $= -2b$.

Substituting these in the expressions for N and S, we find (after all reductions)

$$N = \frac{b^2}{3} . n . \overline{n+1} . \overline{n+2}$$

$$S = \frac{b}{6} . n . \overline{n+1} . \overline{n+2},$$

and the equation $\frac{N}{S} = 2b$ becomes identical. Therefore b is arbitrary. For convenience, make $b=1$. Then the weights for the results of the successive Swings are, n, $2n-2$, $3n-6$, $4n-12$, &c.

The square of the probable error of the final result was found $=e^2\times\frac{N}{S^2}$. Substituting, this becomes $e^2\times\frac{12}{n.\overline{n+1}.\overline{n+2}}$; or the probable error $=e\times\sqrt{\frac{12}{n.\overline{n+1}.\overline{n+2}}}$.

41. It will be instructive to contrast this result with the result obtained on two other suppositions.

First: suppose that the Swings had been continuous, but that there had been no intermediate comparisons of clocks. The probable error of the first comparison being e, and that of the last comparison being also e, the probable error in their combination by subtraction will be $e\sqrt{2}$; and as this applies to n Swings, the probable error on the mean $=\frac{1}{n}e\sqrt{2}=e\sqrt{\frac{2}{n^2}}$. Comparing this with the probable error found above, it appears that the intermediate comparisons have diminished the probable error in the proportion expressed by the fraction $\sqrt{\frac{6n}{\overline{n+1}.\overline{n+2}}}$. When $n=26$, this fraction is $\sqrt{\frac{13}{63}}$, or the weight of the result is increased nearly five-fold by the intermediate comparisons. When $n=15$, the fraction is $\sqrt{\frac{45}{136}}$, or the weight is increased three-fold.

Second: suppose that the Swings had been discontinuous. The probable error in each Swing, found by combining its first and last comparison, would have been $e\sqrt{2}$; and, as the different Swings are strictly independent, the probable error on the mean of all would have been $e\sqrt{\frac{2}{n}}$. Comparing this with our probable error above, it appears that our system has diminished the probable error in the proportion $\sqrt{\frac{6}{\overline{n+1}.\overline{n+2}}}$. When $n=26$, this fraction is $\sqrt{\frac{1}{126}}$, or the weight of the result is increased 126-fold by our system. When $n=15$, the fraction is $\sqrt{\frac{3}{136}}$, or the weight in increased 45-fold.

These contrasts will suffice to show the great advantage of a system of continuous Swings with intermediate comparisons such as has been employed in this experiment. I cannot quit this subject without repeating that my first impression on the advantage of such a system was derived from the representations of Mr. SHEEPSHANKS*, on occasion of the experiments of 1828.

42. In the First and Second Series, $n=26$, and the successive weights are 26, 50, 72, 92, 110, 126, 140, 152, 162, 170, 176, 180, 182, 182, 180, ... 50, 26. In the Third and Fourth Series, $n=15$, and the successive weights are 15, 28, 39, 48, 55,

* Since I commenced drawing up this paper, my valued friend has been snatched away by death; a victim, I believe, to his labours gratuitously undertaken for the formation of the National Standard of Length.

60, 63, 64, 63, ... 15. Combining the separate results by these weights, we obtain the following mean results :—

First Series.

Log. Rate of Pendulum 8 below upon Pendulum 1821 above
$=9\cdot99928536.$

Second Series.

Log. Rate of Pendulum 1821 below upon Pendulum 8 above
$=0\cdot00073691.$

Third Series.

Log. Rate of Pendulum 8 below upon Pendulum 1821 above
$=9\cdot99928584.$

Fourth Series.

Log. Rate of Pendulum 1821 below upon Pendulum 8 above
$=0\cdot00073715.$

43. For ascertaining the probable error *e* I have used the following process.

Let E_0, E_1, E_2, &c., as before, be the actual errors of comparison, estimated by the effect which they produce on a 4-hours' rate, in the 8th decimal place of logarithms; let R_1, R_2, &c. be the successive individual results for the rate of the lower pendulum on the upper pendulum; and let A be the adopted value of that rate. Now if A were rigorously correct, we should have the following equations :—

$$\begin{aligned} E_0 &= E_0 \\ E_1 &= E_0+R_1-A=E_0+R_1-A \\ E_2 &= E_1+R_2-A=E_0+(R_1+R_2)-2A \\ E_3 &= E_2+R_3-A=E_0+(R_1+R_2+R_3)-3A \\ &\text{\&c.} \end{aligned}$$

and, adding all for the First Series, which terminates with R_{26} and E_{26},

$$E_0+E_1+\text{\&c.}+E_{26}=27E_0+26R_1+25R_2+\text{\&c.}+R_{26}-\frac{26\cdot25}{2}A$$

E_0 is yet undetermined. Now the Theory of Probabilities which we have used requires that the chances of positive and negative errors be equal, and therefore that (subject to the irregularities of chance) $E_0+E_1+\text{\&c.}+E_{26}=0$. This gives

$$E_0=\frac{26\cdot25}{2\cdot27}A-\frac{1}{27}(26R_1+25R_2+\text{\&c.}+R_{26});$$

and, substituting this for E_0 in the different expressions above, E_1, E_2, &c. will be formed. Squaring each, forming $\frac{1}{27}$th part of the sum of squares, and multiplying its square root by 0·6745, the probable error is obtained.

The quantity thus obtained is however a little too great. For, the number which we have found for E_0 contains $\frac{26\cdot 25}{2\cdot 27}$ A or 12A nearly; and as the probable error of A is about $\frac{1}{40}e$, the probable error of 12A is about $\frac{3}{10}e$; and therefore we have on the right side of the equation an aggregate of terms whose probable error is $\sqrt{e^2+\frac{9}{100}e^2}$ or $e\left(1+\frac{1}{22}\right)$ nearly. The same is true for E_{26} and those near it. But for E_{13} the factor of A is 0. Thus it will easily be seen that the quantity which we obtain is really $e\left(1+\frac{1}{66}\right)$ nearly. The correction scarcely deserves notice.

44. In this manner the following (uncorrected) values of E_0, E_1, &c. are found; arranged with reference to the Swings to which they relate. It will be remembered that the number 300 represents an error of $0^s{\cdot}1$ in absolute time, very nearly.

First Series.			Second Series.			Third Series.			Fourth Series.		
No. of Swing.	Error of Comparison.		No. of Swing.	Error of Comparison.		No. of Swing.	Error of Comparison.		No. of Swing.	Error of Comparison.	
	+	−		+	−		+	−		+	−
...		454	...		276	...	476		...	211	
1		923	27		501	53	30		68		128
2		414	28	7		54		476	69	254	
3	55		29		82	55	575		70		299
4		220	30		89	56		52	71		266
5	702		31	88		57		209	72	124	
6	1180		32	316		58		353	73	100	
7	925		33		56	59		276	74		234
8	874		34	341		60		74	75		217
9		378	35	121		61		361	76	9	
10		798	36		59	62	374		77	223	
11		651	37		122	63	196		78	57	
12		909	38	191		64		145	79	339	
13	79		39	403		65	136		80	165	
14		322	40		56	66		82	81	277	
15	765		41	94		67	234		82		607
16	707		42	123							
17	672		43	146							
18	517		44	182							
19		296	45		127						
20		108	46		109						
21	439		47	52							
22		392	48	79							
23		1135	49		241						
24	112		50	9							
25		39	51		164						
26	7		52		275						

From these are found the values of the probable error of a single comparison, treating the four series separately:

In Series 1, $e = \pm$ 420.
In Series 2, $e = \pm$ 135.
In Series 3, $e = \pm$ 203.
In Series 4, $e = \pm$ 173.

And hence the probable errors and weights of the mean results are:—

$$\text{For First Series, probable error} = \pm\ 10{\cdot}4,\ \text{weight} = \frac{93}{10000}.$$

$$\text{For Second Series, probable error} = \pm\ 3{\cdot}3,\ \text{weight} = \frac{899}{10000}.$$

$$\text{For Third Series, probable error} = \pm\ 11{\cdot}0,\ \text{weight} = \frac{82}{10000}.$$

$$\text{For Fourth Series, probable error} = \pm\ 9{\cdot}4,\ \text{weight} = \frac{114}{10000}.$$

45. Combining the results of the First and Third Series, with the weights just found, and still adopting as unit, in the probable error, the unity of the 8th decimal of logarithms,

Log. Rate of Pendulum 8 below upon Pendulum 1821 above
$= 9{\cdot}99928558 \pm 7{\cdot}5.$

Combining the results of the Second and Fourth Series,

Log. Rate of Pendulum 1821 below upon Pendulum 8 above
$= 0{\cdot}00073694 \pm 3{\cdot}1.$

And, remarking that Log. $\frac{\text{Gravity below}}{\text{Gravity above}}$ is the sum of these logarithms, we have finally

$$\text{Log.}\ \frac{\text{Gravity below}}{\text{Gravity above}} = 0{\cdot}00002252 \pm 8{\cdot}2,$$

$$\text{or}\quad \frac{\text{Gravity below}}{\text{Gravity above}} = 1{\cdot}00005185 \pm 0{\cdot}00000019;$$

or we may otherwise express it,

Gravity below is greater than Gravity above by $\frac{1}{19286}$ part, with an uncertainty of $\frac{1}{270}$ part of the excess.

The acceleration of a seconds' pendulum below is $2^{s}{\cdot}24$ per day, with an uncertainty of less than $0^{s}{\cdot}01$.

46. But it is to be remarked that this estimate of the amount of uncertainty is obtained from the amount of uncertainty in each mean as deduced separately from the discordances of the individual results in the group contributing to that mean. In comparing the means, we shall see reason to suppose that the ultimate uncertainty is greater. Thus, though the probable errors of the means of Series 1 and 3 are 10·4 and 11·0, the difference of the means is 48: though the probable errors of the means of Series 2 and 4 are 3·3 and 9·4, the difference of the mean is 24. It is likely therefore that some cause of irregularity has occurred, special to each series. The most

probable cause is some trifling error or unsteadiness in the manner of fixing the agate-plane-frames upon the iron stands. It is not a change in either of the pendulums after the Second Series, inasmuch as the value of the mean is altered the same way under the alternation of position of the pendulums. Whatever the cause may have been, the effect is extremely small. There appears to be no reason for altering the concluded ratio of Gravity below to Gravity above. The probable error stated in the last paragraph may be doubled, but I think that there is no sufficient ground for trebling it. The amount of the uncertainty, so increased, is insignificant for the purposes of this experiment.

47. There remains, however, a serious question whether there may have been any difference in the circumstances of the upper and lower pendulums, not included in the corrections applied, which can produce an effect similar to that of a change of gravity. The first point to be considered is, the instability of the mountings. The importance of a very firm foundation was perfectly understood by the able practical men by whom the ground-work was arranged, and particularly by Mr. Arkley; and I conceive all was done which it was possible to do, to make the floors solid. The form of the iron stands is particularly well adapted to firmness. Any tendency to lateral or other movement is counteracted by the endwise resistance of strong straight iron bars. I had at first intended to interchange the iron stands in the middle of the operation, but upon contemplating the mechanical firmness ensured by the plan of their construction, and the exact similarity of the two stands in every respect, I gave up this design; being fully convinced that there might be risk of instability in a change, but that there could scarcely be any sensible absolute instability and (as I believe) no sensible relative instability, in the stands as they were planted. The stands were supported in the same manner at both stations.

The mere determination of the relative rates of the detached pendulum and the clock pendulum, by the method of coincidences, is accurate almost beyond conception. I do not see how it is possible that an error of $0^{s}{\cdot}01$ per day can enter from this cause.

The observers were so evenly interchanged in the upper and lower stations, that no personal peculiarity in the method of reading the thermometers or of taking any other observations can have produced a sensible effect. The following Table shows the aggregate number of turns taken by each:—

Initials of observer's name	C	D	E	P	R	S.
Number of turns above	5	5	4	4	5	5.
Number of turns below	5	2	5	6	5	5.

48. I will now point out the only cause from which, in my judgment, any perceptible error can arise. It was my intention that the temperature of the upper station should be brought, as near as could practically be done, to that of the lower station. In the first week, however, Mr. Dunkin was seized with a sudden and severe illness,

and the transmission of the detailed observations to me was in consequence delayed. An inequality of temperature, which began accidentally, was thus allowed to exist too long to admit of correction. On comparing the mean of all the upper temperatures with the mean of all below, I find that the lower station was warmer than the upper by 7°·13 FAHR. (the upper and lower means being 55°·75 and 62°·88). Therefore, if the adopted coefficient of thermal correction is erroneous, the result of these experiments will be erroneous by the amount of that error on a range of temperature of 7°·13. There may yet be opportunity of verifying the coefficient of temperature-correction.

SECTION VII.—*Measure of the difference of level of the two Pendulum-Stations; survey of the neighbouring country; and computation of the difference of attraction of the Earth's shell on the two pendulums, upon an assumed value of density.*

49. For measuring the depth of the mine, Mr. SIMMS furnished an iron wire 100 feet long, with an attached scale at each end. After the operation, this wire was returned to Mr. SIMMS, and was found to have preserved its length sensibly unaltered. It was used by Mr. ARKLEY to measure the depth, in the following manner. Mr. ARKLEY placed himself on the top of the "Cage" in which the coal-trams are conveyed from the bottom of the shaft to the top, and, when it was lowered 100 feet, an assistant attached himself to the strong wire-rope on which the Cage is suspended. The assistant carried the upper end of the measuring-wire, and in the first instance held it level with the top of the shaft (the steam-engine used for raising or lowering the Cage being stopped); Mr. ARKLEY noted the position of the lower end of the wire (to which a small stretching weight was attached), and fixed a small nail in the brattice opposite to that lower end. Then the Cage was lowered 100 feet; the assistant held the top of the wire to the nail so fixed, and Mr. ARKLEY drove a second nail in the brattice, opposite the lower end; and so on to the bottom; the last fraction of 100 feet being measured with a measuring tape. After this, the levels of the pendulum-stations were referred to the top and bottom of the shaft by the ordinary surveyors' levelling operations.

The following is an abstract of the result:—

	ft.	in.
Depth of the shaft	1263	6
Rise of the floor of the mine from the point of measurement at the bottom of the shaft to the floor of the lower pendulum-station	4	$9\frac{3}{4}$
Fall of the surface from the point of measurement at the top of the shaft to the floor of the upper pendulum-station	2	$7\frac{1}{4}$
Distance between the floors of the upper and lower pendulum-stations .	1256	1

50. For computing the attractions which determine the difference of gravity at the

upper and lower stations; if we refer to the simple investigation in article 2, it will appear desirable to conduct the calculation which is to apply to the earth's irregular form in such a manner as to preserve the characteristics of that simpler investigation as closely as possible. These characteristics are,—

1. That a shell may be traced, whose inner surface passes through the lower station, and whose attraction at that lower station is =0.
2. That the attraction of the same shell at the upper station is the same as if its matter were collected at the centre of the earth.
3. That the attraction of the inclosed nucleus follows the same law, in reference to the difference at the upper and lower stations, as if all its matter were collected at the centre of the earth.

And we are to find how nearly we can approach to these circumstances on the supposition that the earth's constitution is irregular, both in the neighbourhood of Harton and in distant regions.

51. Now if there are sensible irregularities near the upper station, it will be impossible to satisfy the first and second conditions at the same time. For, the demonstration of the evanescence of shell-attraction at the lower station rests upon this: that if chords be drawn through the point L, Plate XI. fig. 1, included within the solid angle aLb, as cLFC, the portion FC must be equal to Lc; and therefore, if in the surface ab (which may be a very minute field) there be an elevation or depression, there must be a corresponding elevation or depression over the whole AB (which will be, in extent, a large continent); and this will disturb the second condition. It will be better, therefore, in the first instance, to give no attention to the local irregularities near the upper station; to assume that the surface there is spherical; to find with this assumption how we can satisfy the three conditions; and afterwards to make allowance for the effect of the irregularities near the upper station.

52. In fig. 2, then, conceive that for some distance on each side of L (say twenty or thirty times the depth of L) the external surface is sensibly spherical; and conceive that at A, B, C, &c. there are local irregularities, perhaps large in extent as compared with the depth of L, but very small as compared with AB in fig. 1. Trace the inner surface DEF by making AD=La, BE=Lb, CF=Lc, &c. These lines, however, are to be made geometrically equal only when the density of the matter is the same as that above L; if the density about A is less than that above L, take the geometrical length AD greater than La in the same proportion. Then the attraction of the shell on the point L will be strictly equal to 0. Moreover, its attraction on the point U will be sensibly the same as if its form were free from irregularities. For, the attractions on U are all *added* together; the irregularities are local and numerous, and are partly additive and partly subtractive; and by hypothesis we have excluded all the irregularities near L or U, which, individually, can be important. And it may be accepted as a universal principle, that when a result is produced by the *addition* of a great number of small components which are liable individually to

small irregularities + or — affecting the ratio or multiplier of each, the sum of all the components will be sensibly free from the effects of these irregularities.

53. In like manner, if we divide the nucleus by planes parallel to the tangent at L or U, as shown by the dotted lines in fig. 2 (or indeed in any other way), the attractions of the slices thus formed are additive, both in their effect on L and in their effect on U. Therefore, for the nucleus generally, by the same reasoning as that above, the effect of irregularities in the outline (and therefore the effect of the irregularities in the outline of the earth, on which these depend) will be, as I conceive, sensibly evanescent. But this does not apply to irregularities in the geological constitution of the earth at a small distance below L; because these irregularities, or rather that one irregularity, may be sensible in proportion to the whole change of attraction between U and L. This is a source of uncertainty from which no experiments made on the earth itself can be perfectly free. We must trust in a great measure to the general regularity of stratification, &c. of the district, for supporting us in the confidence that there is no great disturbance in the law of attractions of the nucleus upon the points U and L.

54. To illustrate in some degree the difference in the attractions and changes of attraction depending on different slices of a sphere, I have supposed a homogeneous sphere divided into twenty slices by equidistant planes parallel to the tangent at L, and have computed (by formulæ easily investigated) the attraction of each slice upon the point L at the surface, and upon a point raised above the surface by $\frac{1}{10}$th part of the radius. Omitting the factor π, the results are as follows :—

No. of slice.	Attraction on point at the surface.	Attraction on point elevated $\frac{1}{10}$ radius.	Decrease by the elevation of the point.
1	·17019	·10401	·06618
2	·14548	·11404	·03144
3	·12941	·10644	·02297
4	·11640	·09811	·01829
5	·10519*	·09002	·01517
6	·09515	·08231	·01284
7	·08600	·07501	·01099
8	·07756	·06808	·00948
9	·06963	·06144	·00819
10	·06217	·05509	·00708
11	·05512	·04901	·00611
12	·04833	·04315	·00518
13	·04191	·03745	·00446
14	·03568	·03207	·00361
15	·02973	·02666	·00307
16	·02391	·02161	·00230
17	·01836	·01655	·00181
18	·01295	·01165	·00130
19	·00766	·00693	·00073
20	·00249	·00232	·00017

* I was not aware till I made this calculation that the plane which bisects the radius drawn to L divides the sphere into two segments whose attractions on L are equal.

These different slices, it may be remarked, correspond to equal surfaces on the sphere; and upon these it is not improbable that the irregularities may mainly depend.

55. In much of the preceding reasoning, it will be remarked, I have tacitly assumed that large continental elevations or large marine depressions, as we find them on the earth, do not interfere materially with the general law of attraction based on the spherical distribution of matter. For the reasons which seem to sustain this assumption, I would refer to a paper by me (printed in the Philosophical Transactions, 1855) on the Attractions of Mountain Masses. It will also be remarked that I have not introduced the consideration of the earth's rotation. I conceive its effects to be extremely insignificant; but the formulæ applying to it are so unmanageable, that I have not pursued it to details.

Considering now that it is sufficiently shown that, on the supposition that the surface in the neighbourhood of U is truly spherical, we may use the method of article 2, with no other uncertainty than that explained in article 53: I shall proceed with the corrections for the inequalities of the surface near U.

56. First, I shall investigate the attraction of the matter included between two horizontal planes, figure 3, upon points U and L in these planes, whose distance or the separation of the planes is equal to the distance UL in figure 2.

Divide the whole of the matter into cylindrical rings, of which UL is the axis: let the internal and external radii of one of these rings be ϱ and $\varrho+\delta\varrho$. Call the azimuth of any part of the ring θ; the end-surface of the prism included between θ and $\theta+\delta\theta$ is $\varrho.\delta\varrho.\delta\theta$. Let z be the vertical ordinate measured upwards from the lower plane; the solid content of the part of the prism included between z and δz is $\varrho\delta\varrho.\delta\theta.\delta z$: its attraction on the point L, supposing its density to be d, is $\frac{d.\rho\delta\rho.\delta\theta.\delta z}{\rho^2+z^2}$; and the resolved part of this, in the vertical direction, is $\frac{d.\rho\delta\rho.\delta\theta.z\delta z}{(\rho^2+z^2)^{\frac{3}{2}}}$. Integrating with respect to z between the limits $z=0$ and $z=c=\text{UL}$, we have $d.\varrho\delta\varrho.\delta\theta.\left(\frac{1}{\rho}-\frac{1}{(\rho^2+c^2)^{\frac{1}{2}}}\right)$. Integrating with respect to θ for the whole circumference, we have $2\pi.d.\left(\delta\varrho-\frac{\rho\delta\rho}{(\rho^2+c^2)^{\frac{1}{2}}}\right)$. Integrating with respect to ϱ, we have $2\pi.d.\{\varrho+c-(\varrho^2+c^2)^{\frac{1}{2}}\}$. This is the attraction upwards on the point L. The attraction downwards on the point U will be the same; and thus the difference of attractions on U and L, estimated in the downwards direction, will be $4\pi.d.\{\varrho+c-(\varrho^2+c^2)^{\frac{1}{2}}\}$.

If the planes be continued without limit, or ϱ be infinite, this expression becomes $4\pi c.d$. Now the attraction of a shell whose thickness is c, computed as in article 2, is 0 for the point at the inner surface of the shell, and $4\pi c.d$ for the point at the outer surface, and therefore the difference is $4\pi cd$. Hence it is indifferent whether we consider the difference of attractions at the upper and lower stations (independent of the change in the attraction of the nucleus caused by the change of distance from it), as produced by a shell of matter, or as produced by the matter between two parallel planes of unlimited extent.

If the extent of the planes be limited, their form being circular, let $\varrho=nc$; then the difference of attractions is $4\pi c.d.\left\{n+1-\left(n+\frac{1}{2n}-\text{ \&c.}\right)\right\}=4\pi c.d.\left\{1-\frac{1}{2n}\right\}$ nearly. If (as in the Harton experiment) $r=\frac{1}{4}$ mile, $\varrho=3$ miles $=12c$ nearly, this $=4\pi c.d\left\{1-\frac{1}{24}\right\}$ nearly. Thus it appears that $\frac{23}{24}$ of the effect is produced by the matter within three miles of the pendulum-stations. It is evident therefore that it is not necessary to give attention to the small inequalities of the ground at any great distance from the pendulum-stations.

57. The inequalities which we have to consider are entirely at the surface, and do not in any case exceed in vertical measure one-tenth of the depth of the mine. They may therefore be considered as being actually at the surface; and, if their horizontal extent is not very great, each may be considered as collected at its centre of gravity. Its effect on the upper station will be 0; its effect on the lower station will be $\frac{c}{(\varrho^2+c^2)^{\frac{3}{2}}}\times d\times$ volume; where ϱ is the horizontal distance of the centre of gravity. An eminence will increase the attraction upwards, and a depression will diminish it. But as our only object is to find the difference of attractions on the two stations, we may estimate the whole, with changed sign, as an effect on the upper station; then an eminence will increase the attraction downwards, and a depression will diminish it.

There is however one depression which it is desirable to consider in a different way, namely that of the sea. The depth of the sea itself is less important than the depression below the table-land, which continues with little change of level to the edge of the cliffs; and a sufficiently accurate estimate may be formed of the measure of depression (including the effect of the attraction of the water) considered as uniform. The line of cliff may be considered as straight. Let a be the distance from the pendulum-station to the straight line of cliff, measured perpendicularly to that line; b the depth of the depression; let x be parallel to a, and y parallel to the line of cliff. The matter $d\times\delta x\times\delta y\times b$ is at the distance $(x^2+y^2)^{\frac{1}{2}}$ from the upper station, and therefore its vertical attraction on the lower station is $\frac{d.b.c.\delta x.\delta y}{(x^2+y^2+c^2)^{\frac{3}{2}}}$, or $\frac{d.b.c.\delta x.\delta y}{(x^2+y^2)^{\frac{3}{2}}}$ nearly. Integrating first with respect to y, from $-\infty$ to $+\infty$, we have $\frac{2d.b.c.\delta x}{x^2}$. Integrating then with respect to x, from a to ∞, we have $\frac{2d.b.c}{a}$.

These formulæ will suffice for our purpose. It is only necessary further to remark, that, as the unit of measure is absolutely arbitrary, and as the numerical and graphical operations are a little facilitated by using for unit the "depth of the mine," I have, in all the subsequent calculations, adopted that quantity (1256 feet) as unit of measure. In all the preceding formulæ, c must now be made $=1$. The earth's radius corresponding to Harton is then 16621·7, its attraction at the lower station is $\frac{4\pi}{3}\times 16621{\cdot}7\times D$, or $69625\times D$, and at the upper station $\left\{69625-\frac{8\pi}{3}\right\}\times D$. The attraction of the shell upon the upper station, or the difference of attractions of the indefinite horizontal stratum upon the upper and lower stations, is $4\pi\times d$. The formula applicable to the depression of the sea will be $\frac{2b}{a}\times d$; and that applicable to any other elevation or depression will be $\frac{\text{volume}}{(\rho^2+1)^{\frac{3}{2}}}\times d$; where all linear measures are to be referred to the mine-depth as unit.

58. On my making known that information on the inequalities of the ground would be required for the final calculations, the Mayor and Corporation of South Shields directed that the necessary surveys should be made, and the execution of this work was entrusted to CHRISTOPHER THOMPSON, Esq., Surveyor for the Corporation. This gentleman entered fully into my views, and, after the proper surveys for elevation, furnished me with a map extending about three miles in all directions round Harton, with the elevations above high water in feet marked at numerous points. I found that a line might be drawn, nearly ten depths distant from the upper station, touching the cliffs of Tynemouth and the cliffs south-east of Harton, and ranging for some distance along the coast of Durham. I therefore drew a line parallel to this at ten depths distance from the upper station, and divided the whole country into squares (with sides of one depth each) whose sides were parallel and perpendicular to this. These squares I grouped as appeared most convenient, as will be seen in the Map, Plate XII. fig. 4 (the principal object being to secure a proper representation of Jarrow Slake and the Valley of the Tyne), and adopted for each group the elevation in feet above high water which Mr. THOMPSON's elevations of special points suggested. The elevation of the upper pendulum-station was 74 feet. Consequently, the vertical measure which was to be used for computing the "volume" in the formula above was $\frac{\text{Elevation}-74}{1256}$. Of this formula, a small table was prepared. The quantity ϱ was measured graphically from the map, and $\sqrt{\varrho^2+1}$ was formed graphically from it.

59. Adopting for the sea, so far as its effects are principally sensible (and conceiving the water replaced by an equivalent quantity of ground with surface lower than the sea), the elevation -15 feet, to which corresponds

$$b=\frac{-15-74}{1256}=\frac{-89}{1256}=-{\cdot}07087,$$

the factor of d applicable to the depression of the sea beyond the straight line at distance 10 is

$$\frac{-2\times{\cdot}07087}{10}=-{\cdot}014174.$$

The elements for computing the effect of the other inequalities, and the factors of d, are as follows; that for 19* being inserted by estimation:—

No.	Elevation in feet above high water.	Surface.	ρ.	Attraction.		No.	Elevation in feet above high water.	Surface.	ρ.	Attraction.	
				+	−					+	−
1	− 15	......	......	......	·014174	23	15	3	2·6	......	·006421
2	0	9	10·7	......	·000427	24	60	4	1·7	......	·005920
3	0	8	9·5	......	·000541	25	55	4	1·7	......	·008021
4	5	7	10·0	......	·000379	26	80	4	1·7	·002537	
5	5	6	9·8	......	·000346	27	74	4	1·7	·000000	
6	30	6	8·3	......	·000361	28	90	8	3·0	·003292	
7	70	12	7·0	......	·000108	29	90	12	5·5	·000866	
8	80	7	6·9	·000099	......	30	90	12	8·5	·000244	
9	130	12	7·8	·001102	......	31	0	6	16·4	......	·000080
10	125	10	9·7	·000438	......	32	0	4	15·3	......	·000065
11	0	4	7·5	......	·000543	33	0	12	11·0	......	·000524
12	15	4	5·8	......	·000925	34	− 5	5	7·0	......	·000890
13	5	2	5·2	......	·000743	35	30	4	9·7	......	·000151
14	50	6	3·9	......	·001739	36	40	20	6·8	......	·001669
15	80	6	3·7	·000514	......	37	20	2	4·0	......	·001229
16	100	16	4·5	·003381	......	38	74	25	4·4	·000000	
17	120	12	7·0	·001244	......	39	105	16	6·5	·001387	
18	120	24	10·5	·000748	......	40	90	21	8·5	·000426	
19	− 15	50	18·0	......	·000605	41	140	16	9·3	·001029	
19*				......	·000600	42	100	24	10·4	·000435	
20	15	2	7·9	......	·000186	43	125	16	12·0	·000372	
21	0	2	6·6	......	·000395	44	120	10	11·0	·000272	
22	− 10	9	4·5	......	·006143						

The sum of all the factors for the effects of inequality is —·044799.

As the attraction of the shell is $12·566368 \times d$, and as the effects of inequality amount to $-·044799 \times d$, the complete effect of the ground above the lower station is $12·521569 \times d$.

60. We are now in a position to obtain a result requiring only for its complete numerical exhibition a knowledge of the specific gravities of the mine-rocks. The formula for gravity at the lower station has been found to be

$$69625 \times \mathrm{D},$$

and that at the upper station

$$(69625-8·3776) \times \mathrm{D}+12·5216 \times d.$$

Hence $$\frac{\text{Gravity below}}{\text{Gravity above}}=1+0·00012032-0·00017984 \times \frac{d}{\mathrm{D}}.$$

But the experiments gave (see article 45)

$$\frac{\text{Gravity below}}{\text{Gravity above}}=1·00005185 \pm ·00000019.$$

Hence $$0·00006847 \pm ·00000019=0·00017984 \times \frac{d}{\mathrm{D}};$$

or $$\frac{\mathrm{D}}{d}=2·6266 \pm ·0073,$$

where (as has been explained in article 46) the last term ought probably to be doubled, to give the uncertainty depending on the pendulum-experiments only.

SECTION VIII.—*Estimation of the Mean Specific Gravity of the Surface Rocks; and final conclusion on the Mean Density of the Earth, as determined from the Pendulum Experiments.*

61. By the kindness of Mr. ARKLEY, I was furnished with a complete account of the strata passed through in sinking the Harton shaft, and with specimens of a great number of the rocks. These were submitted to Professor W. H. MILLER, who with great labour determined their specific gravities. I place below the entire catalogue and measure of the strata, and the results at which Professor MILLER arrived. Professor MILLER remarks that there is inevitable uncertainty in the estimation of the weight of those specimens which were drawn from beds described as containing water.

No.	Description of Rock.	Thickness. ft.	Thickness. in.	Specific Gravity.	Temperature of water, Centigrade. °
1	Soil	1	0		
2	Yellow Clay	6	4½		
3	Blue laminated Clay	7	0		
4	Gravelly Clay	69	2		
5	Red Clay (resting on the Stone Head)	10	0		
6	Soft blue Shale, and a little Water	7	0		
7	Coal		3		
8	Dark Grey Shale	7	6	2·5698	15·9
9	Coal		3		
10	Dark Grey Shale	12	3	2·5273	16·4
11	Coal and a little Water		4		
12	Strong dark Grey Shale mixed with Sandstone	9	2		
13	Strong Grey Sandstone and Water	7	10	2·5341	8·8
14	Grey Shale	18	0	2·6170	9·2
15	Bituminous Shale		5		
16	Coal	1	0		
17	Thill or Grey Shale	4	2		
18	Sandstone Girdle		10		
19	Grey Shale, Sandstone Girdles, and Water	14	2		
20	White Sandstone Girdle and Water	1	2		
21	Grey Shale	1	6		
22	White Sandstone Girdle and Water	3	5		
23	Bituminous Shale	2	9		
24	Coal		4		
25	Thill or Grey Shale	10	0	2·5325	9·7
26	Grey Sandstone	3	2	2·6325	9·5
27	Grey Shale	2	6		
28	Grey Sandstone with partings of Bituminous Shale	2	8		
29	Blue Shale	5	8		
30	Coal	1	2		
31	Dark Grey Shale	4	6		
32	Grey Shale	10	0		
33	Coal	1	2		
34	Band of Grey Shale		4		
35	Coal		6		
36	Sandstone with very hard Girdles	7	0	2·8072	17·25

TABLE (continued).

No.	Description of Rock.	Thickness. ft.	in.	Specific Gravity.	Temperature of water, Centigrade.
		ft.	in.		°
37	Grey Shale and Water	3	4		
38	Coal		6		
39	Bituminous Shale		3		
40	Coal		3		
41	Grey Shale with Sandstone Girdles	8	4		
42	Coal		3		
43	Grey Shale	5	2		
44	Coal		2		
45	Grey Shale with thin Sandstone Girdles and Water	23	7	2·5525	15·8
46	Coal, Brassey Band, and Coal	2	10	1·3117	16·1
47	Grey Shale	4	0	2·5716	10·0
48	Grey Shale, Sandstone Girdles, and Water	15	1	2·5687	10·3
49	Sandstone and Water	45	0	2·6695	8·8
50	Dark Grey Shale	12	0	2·5962	9·9
51	Coal	1	8	1·3474	7·7
52	Grey Shale or Thill	4	0		
53	Grey Sandstone and Water	5	2		
54	White Sandstone with Shale partings and Water	116	8	2·8016	16·0
55	Dark Grey Shale	16	3	2·6132	16·4
56	Coal		8		
57	Dark Grey Shale	16	3	2·6233	4·8
58	Coal		5		
59	Brown Shale		6		
60	Grey Sandstone with Shale partings and salt water, emitting Carburetted Hydrogen Gas	7	3½	2·5410	16·9
61	Red and White Sandstone, porous and wet	53	8½	2·8163	8·4
62	Coal mixed with hard Bituminous Shale		3	1·7010	8·4
63	Soft Grey Shale	7	0	2·5184	9·5
64	Strong Bituminous Shale mixed with Nodules of Clay Ironstone	1	5	2·2587	9·5
65	Coal		2½		
66	Strong light-coloured Grey Sandstone with Shale partings and very hard Girdles	10	0	2·6595	9·1
67	Dark Grey Shale	10	7	2·5734	17·5
68	Coal; Bituminous Band; Coal; Bituminous Band; Coal	1	8	1·4839	17·5
69	Hard Grey Shale with very hard Sandstone Girdles	27	4	2·6223	7·7
70	Dark Grey Shale with Girdles of Clay Ironstone	15	10	2·5416	8·9
71	Foul Coal		5		
72	Dark Grey Shale	2	6	2·7073	18·5
73	Hard dark Grey Shale with Sandstone Girdles	12	0	2·2660	9·5
74	Coal		5		
75	Grey Shale	1	8		
76	White Sandstone with very hard Girdles and Water	24	11	2·5334	9·0
77	Coal 0 ft. 4 in., Slaty Coal 1 ft. 1 in.=1 ft. 5 in.	1	5		
78	Grey Shale	2	7		
79	Coal		6		
80	Dark Grey Shale with Sandstone Girdles and Water	14	1	2·6434	9·3
81	White Sandstone with Shale partings	13	11½	2·5613	7·8
82	Grey Shale	1	4		
83	Coal, strong and good	2	6		
84	Splint Coal		6		
85	Grey Shale	1	6		
86	Strong White Sandstone, very hard in places, and water	43	8½	2·4969	15·5
87	Bituminous Shale		3½		

TABLE (continued).

No.	Description of Rock.	Thickness. ft.	Thickness. in.	Specific Gravity.	Temperature of water, Centigrade. °
88	Black Slaty Coal	1	5½		
89	Grey Shale with Sandstone Girdles	15	8		
90	Bituminous Shale with Clay Ironstone Girdles	3	0		
91	Coal		1½		
92	Dark Grey Shale with Sandstone Girdles	16	1		
93	Hard White Sandstone with frequently hard Shale and small Ironstone Nodules very coarse, and a little Water	64	7	2·5483	8·5
94	Dark Blue Shale with Nodules and Ironstone Girdles	6	11	2·5178	9·5
95	Grey Shale with Sandstone Girdles	6	0		
96	Bituminous Shale	7	5	2·5209	17·1
97	Coal, strong and good		6	1·2956	18·1
98	Grey Shale with Sandstone Girdles	21	0		
99	White Sandstone with a little Water	19	0	2·3250	
100	Grey Shale with Ironstone Girdles	6	0		
101	Coal	1	2		
102	Bituminous Shale		1	2·3942	4·8
103	Coal		2½		
104	Bituminous Shale	3	0	2·6000	9·1
105	Bituminous Shale mixed with Coal pipes		6		
106	Grey Shale	5	6		
107	Hard Shale	21	0	2·6066	9·6
108	Grey Shale with Sandstone Girdles	27	0	2·6641	7·3
109	Strong Grey Sandstone with Shale partings, and very hard Sandstone Girdles with a little Water	40	5	2·6146	9·6
110	Blue Shale Girdle	2	0½		
111	White Sandstone Girdle with Shale partings	2	3		
112	Coal	1	10		
113	Grey Shale with Ironstone Nodules	4	0		
114	Strong Grey Shale with Sandstone Girdles	10	0		
115	Strong dark Grey Shale with Sandstone Girdles	14	0		
116	Splint Coal		5		
117	Grey Shale with Sandstone Girdles	10	0		
118	Strong White Sandstone with very hard Girdles	12	0	2·6481	15·2
119	Very hard Sandstone	8	0	2·6763	7·7
120	Hard Grey Shale with Sandstone and Ironstone Girdles	33	1	2·6021	7·4
121	Black Slaty Bituminous Shale	1	0		
122	Coal		9		
123	Bituminous Shale		3		
124	Shale	4	0		
125	Hard Grey Shale with very hard Sandstone Girdles	7	9		
126	Coal		3		
127	Hard Grey Shale with mild and also very hard Sandstone Girdles	25	3	2·8178	5·3
128	Coal		8½		
129	Bituminous Shale	3	2		
130	Coal		3½		
131	Shale	4	2	2·5591	16·8
132	Bituminous Shale		5½		
133	Splinty Coal		2½		
134	Grey Shale	1	2		
135	Dark Grey Shale with Ironstone Girdles	6	2		
136	Grey Shale with Ironstone Girdles	11	3		
137	Grey Shale with Shale partings and very hard Sandstone Girdles	15	0		
138	Dark Grey Shale with Ironstone Girdles	8	0		
139	White Sandstone with very hard Girdles	5	6		
140	Grey Sandstone with Shale partings and very hard Girdles	21	1½	2·6146	8·3
141	Bituminous Shale		4		
142	Top Coal, Splint, Bottom Coal } Bensham Coal Seam.	6	0	1·2869	8·7

62. Without pretending to extreme accuracy, we may consider the thickness and specific gravity of the beds as represented, with sufficient accuracy for the present purpose, by the following numbers:—

1211 feet of rocky and shaly beds . .	specific gravity 2·56
30 feet of coaly beds	specific gravity 1·43
15 feet completely worked out . .	specific gravity 0·00

From these the mean specific gravity is found to be 2·50.

63. Substituting this for d in the expression at the end of the last Section, we get for the mean density of the Earth, as determined from these experiments,

$$6{\cdot}566 \pm {\cdot}0182,$$

where the last term (expressing the probable error) ought to be at least doubled.

64. The value thus obtained is much larger than that obtained from the Schehallien experiment, and considerably larger than the mean found by BAILY from the torsion-rod experiments. It is extremely difficult to assign with precision the causes or the measures of the errors of any of these determinations; and I shall content myself with expressing my opinion, that the value now presented is entitled to compete with the others, on, at least, equal terms.

XV. *Supplement to the "Account of Pendulum Experiments undertaken in the Harton Colliery;" being an Account of Experiments undertaken to determine the Correction for the Temperature of the Pendulum. By* G. B. AIRY, *Esq., Astronomer Royal.*

Received February 13,—Read March 6, 1856.

SECTION IX.—*Introductory and Historical.*

65. IN order to remove, as far as possible, the slight uncertainty in the result of the Harton Experiment depending on the mean difference of temperature of the Pendulums, I again borrowed the two Pendulums and the Clock SHELTON from the Royal Society, and prepared them for use in the following manner. Two rooms were selected at the Royal Observatory, which appeared well adapted for a series of pendulum experiments. One is the Laundry of the Dwelling House; a room on the basement story, almost entirely sunk below the level of the Lawn and Front Court, but bordered on two sides by a dry area; with stone floor laid immediately upon the solid ground. The chimney-opening and other crevices were carefully stopped up, and a hot-air-stove with funnel leading into the chimney was planted in the room; by means of this stove the temperature of the room could be maintained with reasonable steadiness nearly to 90° FAHR. The Pendulum 8 with Clock EARNSHAW were mounted in the usual way in this room. A screen was placed between the stove and the pendulum, which effectually prevented the pendulum and its thermometers from receiving any sensible heat by radiation. The other room is the Lower Record Room, a room with stone floor laid (except at its centre) upon the solid ground, warmed by a heating apparatus below the centre of the floor. The Pendulum 1821 with Clock SHELTON were mounted in one side of this room. As a hot-air-pipe rises in the centre of the room, this pipe also was screened to prevent unfair radiation on the pendulum, &c.; although here the necessity for such caution was far less pressing than in the Laundry.

66. The plan proposed for operations was; to compare the clocks, by carrying a Solar Chronometer from one to the other, and observing their readings at the time of coincidence of beats (a method which admits of extreme accuracy when the clocks are so near together that the chronometer can be carried without risk of disturbance); to divide the operations into Four Series, numbered in sequence Fifth, Sixth, Seventh, Eighth; to make no alteration whatever in the pendulums, nor even to dismount them, between one series and another; but to make Pendulum 1821 hot and 8 cool in the Fifth and Seventh Series, and Pendulum 8 hot and 1821 cool in the Sixth and Eighth Series; and through the extent of each Series to keep up the Swings in

incessant sequence, as in the observations at Harton. The uncontrolled conduct of the experiments was entrusted to Messrs. DUNKIN and ELLIS; and I am confident it will be found that they are inferior to none that have ever been made.

In all cases, the fires for warming the rooms were lighted nearly twenty-four hours before the observations began.

67. When the pendulum-boxes were opened at Greenwich, two of the four thermometers which had been used at Harton were missing, and all attempts to trace them have hitherto failed. Two thermometers belonging to the Royal Observatory (one by THOMAS JONES and one by TROUGHTON and SIMMS) were substituted for them. It would have been desirable to use the very same thermometers which were used in the Harton experiment; but it is probable that, with the considerable range of temperature used in these experiments, the accidental differences between two thermometers made by makers of repute will not produce any sensible effect on the result.

68. The following is the journal of operations :—

Fifth Series. Pendulum 1821 in heat.

1856. Jan. 1.—Observations commenced a little before noon. Mr. DUNKIN observed Pendulum 1821 from the beginning of Swing 83 to the beginning of Swing 85. Mr. ELLIS observed Pendulum 8 from the beginning of Swing 83, and 1821 from the end of Swing 85, to the beginning of Swing 87.

Jan. 2.—Mr. DUNKIN observed 8 from the end of Swing 87 to the beginning of Swing 89; Mr. ELLIS observed 8 from the end of Swing 89 to the beginning of Swing 91; and Mr. ELLIS observed it from the end of Swing 91 to the end of Swing 92. Mr. DUNKIN observed 1821 from the end of Swing 87 to the end of Swing 92, nearly two hours after midnight. This closed the Fifth Series.

Sixth Series. Pendulum 8 in heat.

Jan. 4.—Commencing about $1\frac{1}{2}$ hour before noon. Mr. DUNKIN observed 1821 from the beginning of Swing 93 to the beginning of Swing 97. Mr. ELLIS observed 8 from the beginning of 93 to the beginning of 95; Mr. DUNKIN from the end of 95 to the beginning of 97.

Jan. 5.—Mr. ELLIS observed 1821 from the end of 97 to the beginning of 99; Mr. DUNKIN from the end of 99 to the beginning of 102; Mr. ELLIS at the end of 102. Mr. ELLIS observed 8 from the end of 97 to the end of 102, about $1\frac{1}{2}$ hour after midnight. This closed the Sixth Series.

Seventh Series. Pendulum 1821 in heat.

1856. Jan. 8.—Commencing about 2 hours before noon. Mr. DUNKIN observed 1821 from the beginning of 103 to the beginning of 105; and Mr. ELLIS from the end of 105 to the beginning of 107. Mr. ELLIS observed 8 from the beginning of 103 to the beginning of 107.

Jan. 9.—Mr. DUNKIN observed 1821 from the end of 107 to the end of 112. Mr. DUNKIN observed 8 from the end of 107 to the beginning of 109; Mr. ELLIS from the end of 109 to the beginning of 111; and Mr. DUNKIN from the end of 111 to the end of 112, about 1½ hour after midnight. This closed the Seventh Series.

Eighth Series. Pendulum 8 in heat.

Jan. 11.—Mr. DUNKIN observed 1821 from the beginning of 113 to the beginning of 117. Mr. ELLIS observed 8 from the beginning of 113 to the beginning of 115; Mr. DUNKIN from the end of 115 to the beginning of 117.

Jan. 12.—Mr. ELLIS observed 1821 from the end of 117 to the end of 118; Mr. DUNKIN from the beginning of 119 to the beginning of 121; and Mr. ELLIS from the end of 121 to the end of 122. Mr. ELLIS observed 8 from the end of 117 to the end of 122, about 2 hours after midnight. This terminated the Eighth Series, and the whole operation.

SECTION X.—*Comparisons of Clocks.*

69. The clocks were compared by means of a Solar Chronometer, sometimes three times and sometimes four times at each interruption of Swings. When there are three comparisons, all are made by one observer; when there are four, they are usually (but not in every case) made by two observers. The discordance of the different comparisons in each group does not exceed $0^{s}{\cdot}01$ or $0^{s}{\cdot}02$.

The following Table contains the Mean of each group of comparisons, and the computation of the Log. Rate of SHELTON upon EARNSHAW.

No. of Swing.	Approximate Time of Comparison.	Number of Comparisons.	Mean of Times by SHELTON.	Mean of Times by EARNSHAW.	Interval by SHELTON.	Interval by EARNSHAW.	Rate $\frac{\text{SHELTON}}{\text{EARNSHAW}}$.	Log. Rate $\frac{\text{SHELTON}}{\text{EARNSHAW}}$.
	1856. Jan. h		h m s	h m s	h m s	h m s		
	0. 23	4	6 5 14·00	21 46 41·87				
83					3 11 48·25	3 11 29·47	1·0016345	0·00070928
	1. 2	4	9 17 2·25	0 58 11·34				
84					4 7 39·50	4 7 15·52	1·0016164	0·00070142
	1. 6	4	13 24 41·75	5 5 26·86				
85					4 30 40·92	4 30 14·58	1·0016245	0·00070495
	1. 10	3	17 55 22·67	9 35 41·44				
86					4 24 11·66	4 23 46·18	1·0016100	0·00069865
	1. 15	3	22 19 34·33	13 59 27·62				
87					3 32 26·34	3 32 5·72	1·0016203	0·00070312
	1. 18	3	1 52 0·67	17 31 33·32				
88					3 58 44·00	3 58 20·91	1·0016146	0·00070065
	1. 22	3	5 50 44·67	21 29 54·23				
89					3 42 9·58	3 41 48·11	1·0016133	0·00070008
	2. 2	4	9 32 54·25	1 11 42·34				
90					3 58 2·00	3 57 39·00	1·0016130	0·00069995
	2. 6	4	13 30 56·25	5 9 21·34				
91					4 22 3·42	4 21 38·06	1·0016155	0·00070104
	2. 10	3	17 52 59·67	9 30 59·40				
92					3 36 56·83	3 36 35·90	1·0016105	0·00069887
	2. 14	4	21 29 56·50	13 7 35·30				
	3. 22	4	6 5 12·00	21 39 43·29				
93					3 21 39·00	3 21 19·75	1·0015936	0·00069154
	4. 2	4	9 26 51·00	1 1 3·04				
94					4 13 24·75	4 13 0·56	1·0015935	0·00069150
	4. 6	4	13 40 15·75	5 14 3·60				
95					4 0 22·92	3 59 59·83	1·0016035	0·00069584
	4. 10	3	17 40 38·67	9 14 3·43				
96					3 53 35·00	3 53 12·42	1·0016138	0·00070030
	4. 14	3	21 34 13·67	13 7 15·85				
97					4 40 26·33	4 39 59·23	1·0016131	0·00069999
	4. 18	3	2 14 40·00	17 47 15·08				
98					3 45 54·67	3 45 32·73	1·0016212	0·00070351
	4. 22	3	6 0 34·67	21 32 47·81				
99					3 37 2·83	3 36 41·77	1·0016198	0·00070291
	5. 2	4	9 37 37·50	1 9 29·58				
100					4 4 8·50	4 3 44·81	1·0016191	0·00070260
	5. 6	4	13 41 46·00	5 13 14·39				
101					4 9 8·00	4 8 43·81	1·0016209	0·00070338
	5. 10	4	17 50 54·00	9 21 58·20				
102					4 12 17·75	4 11 53·26	1·0016205	0·00070321
	5. 14	4	22 3 11·75	13 33 51·46				
	7. 22	4	5 48 34·50	21 18 12·38				
103					3 37 41·00	3 37 19·62	1·0016396	0·00071149
	8. 2	4	9 26 15·50	0 55 32·00				
104					4 19 3·25	4 18 37·71	1·0016458	0·00071418
	8. 6	4	13 45 18·75	5 14 9·71				
105					4 44 9·25	4 43 41·31	1·0016415	0·00071231
	8. 10	3	18 29 28·00	9 57 51·02				
106					3 39 8·33	3 38 46·78	1·0016417	0·00071239
	8. 14	3	22 8 36·33	13 36 37·80				
107					4 12 2·34	4 11 37·53	1·0016433	0·00071309
	8. 18	3	2 20 38·67	17 48 15·33				
108					3 43 52·66	3 43 30·66	1·0016405	0·00071188
	8. 22	3	6 4 31·33	21 31 45·99				
109					3 38 56·17	3 38 34·69	1·0016378	0·00071071
	9. 2	4	9 43 27·50	1 10 20·68				
110					4 9 18·00	4 8 53·52	1·0016393	0·00071136
	9. 6	4	13 52 45·50	5 19 14·20				
111					4 1 12·17	4 0 48·49	1·0016390	0·00071123
	9. 10	3	17 53 57·67	9 20 2·69				
112					4 7 8·08	4 6 43·89	1·0016341	0·00070910
	9. 14	4	22 1 5·75	13 26 46·58				
	10. 22	4	5 56 22·50	21 18 57·71				
113					3 58 47·00	3 58 23·91	1·0016143	0·00070052
	11. 2	4	9 55 9·50	1 17 21·62				
114					4 6 8·00	4 5 44·14	1·0016183	0·00070225
	11. 6	4	14 1 17·50	5 23 5·76				
115					4 4 32·17	4 4 8·39	1·0016234	0·00070446
	11. 10	3	18 5 49·67	9 27 14·15				
116					3 40 25·00	3 40 3·57	1·0016230	0·00070429
	11. 14	3	21 46 14·67	13 7 17·72				
117					5 1 35·00	5 1 5·65	1·0016247	0·00070503
	11. 18	3	2 47 49·67	18 8 23·37				
118					3 52 38·00	3 52 15·35	1·0016254	0·00070533
	11. 22	3	6 40 27·67	22 0 38·72				
119					3 15 40·08	3 15 21·02	1·0016262	0·00070568
	12. 2	4	9 56 7·75	1 15 59·74				
120					4 14 49·25	4 14 24·41	1·0016273	0·00070616
	12. 6	4	14 10 57·00	5 30 24·15				
121					4 55 24·00	4 54 55·10	1·0016322	0·00070828
	12. 11	3	19 6 21·00	10 25 19·25				
122					3 46 3·00	3 45 40·97	1·0016270	0·00070603
	12. 15	4	22 52 24·00	14 11 0·22				

SECTION XI.—*Auxiliary Tables for the reduction of the observations of Coincidences.*

70. The range of the Intervals of Coincidences in these experiments is not included in the tables given in Section III. The table of log $\frac{n-2}{n}$ which is placed below includes all that are required here. The method of using it is the same as in the preceding Series.

The tables used before for the Correction for Arc were sufficient for the present experiments.

The range of temperature in the experiments now made greatly exceeded that for the experiments in the mine. In extending a table, so as to include an increased range of physical conditions, there will always be some doubt as to the propriety of selection of that mathematical function which is assumed to increase proportionally with the indications of the physical measurer. I have here assumed that the changes of the logarithms of the temperature-correction are simply proportional to the changes of thermometer-reading. The table below, therefore, is formed from the table in Section III. by simple addition and subtraction of a constant number.

The table used for barometrical correction is the same which was used before.

n.	Log. $\frac{n-2}{n}$.	n.	Log. $\frac{n-2}{n}$.	n.	Log. $\frac{n-2}{n}$.	n.	Log. $\frac{n-2}{n}$.	n.	Log. $\frac{n-2}{n}$.
269·5	9·99676502	271·5	9·99678894	273·5	9·99681250	277·8	9·99686202	279·8	9·99688454
·6	6622	·6	9012			278·9	6315	280·9	8566
·7	6742	·7	9130	276·0	9·99684148	·0	6429	·0	8677
·8	6863	·8	9249	·1	4263	·1	6542	·1	8788
·9	6983	·9	9367	·2	4377	·2	6655	·2	8899
270·0	7103	272·0	9486	·3	4492	·3	6768	·3	9011
·1	7222	·1	9604	·4	4606	·4	6881	·4	9122
·2	7342	·2	9722	·5	4721	·5	6994	·5	9233
·3	7462	·3	9840	·6	4835	·6	7107	·6	9344
·4	7582	·4	9·99679958	·7	4949	·7	7219	·7	9455
·5	7702	·5	9·99680076	·8	5064	·8	7332	·8	9566
·6	7821	·6	0194	·9	5178	·9	7444	·9	9677
·7	7940	·7	0311	277·0	5293	279·0	7557	281·0	9788
·8	8060	·8	0429	·1	5407	·1	7669	·1	9·99689899
·9	8179	·9	0546	·2	5521	·2	7781	·2	9·99690009
271·0	8299	273·0	0664	·3	5635	·3	7893	·3	0119
·1	8418	·1	0781	·4	5748	·4	8006	·4	0230
·2	8537	·2	0898	·5	5862	·5	8118	·5	0341
·3	8656	·3	1016	·6	5975	·6	8230	·6	0451
·4	8775	·4	1133	·7	6088	·7	8342	·7	0562

Thermometer.	Correction to Log. Rate.	Thermometer.	Correction to Log. Rate.	Thermometer.	Correction to Log. Rate.
40°·0	9·99997937	50°·5	0·00000104	88°·0	0·00007840
·5	8040	51·0	0207	·5	7943
41·0	8144	·5	0310	89·0	8046
·5	8247	52·0	0413	·5	8149
42·0	8350	·5	0516	90·0	8252
·5	8453	53·0	0619	·5	8355
43·0	8556	·5	0722	91·0	8458
·5	8659	54·0	0825	·5	8561
44·0	8762	·5	0928	92·0	8664
·5	8865	55·0	1032	·5	8767
45·0	8968			93·0	8870
·5	9071	83·0	0·00006808	·5	8973
46·0	9175	·5	6912	94·0	9076
·5	9278	84·0	7015	·5	9179
47·0	9381	·5	7118	95·0	9282
·5	9484	85·0	7221	·5	9385
48·0	9587	·5	7324	96·0	9488
·5	9690	86·0	7428	·5	9591
49·0	9793	·5	7531	97·0	9694
·5	9·99999897	87·0	7634	·5	9797
50·0	0·00000000	·5	7737	98·0	9900

By the application of the numbers from these two tables, together with those for Arc and Barometer, in the same manner as in the Harton Experiments, there is obtained the "Corrected Log. Rate of Pendulum upon Clock" referred to Barometer 0·000 and Thermometer 50°; but affected by the full amount of the errors of the table for corrections depending on the Thermometer.

SECTION XII.—*Abstract of the Computations of the Rate of each detached Pendulum upon its Clock.*

71. The whole of the reductions, for both stations, are included in the following tables.

Pendulum 1821. Fifth Series.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Interval.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rate of Pendulum 1821 upon SHELTON.
				s			°	in.	
83	3	3	31	272·39	1·67	0·71	84·36	29·775	9·99692466
84	3	3	46	272·81	1·92	0·54	85·03	29·750	3098
85	3	3	48	273·05	2·08	0·54	83·95	29·705	3178
86	3	3	45	273·10	2·09	0·59	84·49	29·645	3351
87	3	3	32	273·01	1·85	0·77	85·26	29·605	3401
88	3	3	38	273·24	2·12	0·75	85·78	29·585	3818
89	3	3	34	273·23	1·97	0·77	84·71	29·525	3553
90	3	3	44	273·44	2·06	0·61	83·31	29·460	3474
91	3	3	48	273·10	1·87	0·51	84·98	29·435	3356
92	3	3	33	273·16	1·75	0·71	85·15	29·430	3487

Pendulum 1821. Sixth Series.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Interval.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rate of Pendulum 1821 upon SHELTON.
93	3	3	35	279·14	2·01	0·74	53·36	29·570	9·99693872
94	3	3	47	278·89	2·02	0·54	54·41	29·545	3754
95	3	3	43	279·04	1·93	0·57	53·38	29·535	3701
96	3	3	37	279·00	2·22	0·78	53·26	29·515	3739
97	3	3	46	279·09	2·05	0·57	53·25	29·465	3738
98	3	3	31	278·96	2·16	0·89	52·91	29·430	3635
99	3	3	32	278·98	2·04	0·82	52·71	29·400	3562
100	3	3	45	279·02	1·94	0·54	52·85	29·370	3533
101	3	3	44	279·21	1·95	0·58	52·35	29·355	3653
102	3	3	43	279·34	2·11	0·63	51·91	29·310	3744

Pendulum 1821. Seventh Series.

No. of Swing.	Number of Coincidences. Beginning.	Number of Coincidences. End.	Number of Intervals.	Mean Interval.	Arc of Vibration. Beginning.	Arc of Vibration. End.	Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rate of Pendulum 1821 upon SHELTON.
103	3	3	41	272·22	2·08	0·71	89·48	28·970	9·99693256
104	3	3	50	271·15	2·22	0·59	93·21	28·985	2754
105	3	3	54	270·83	2·02	0·46	95·60	29·010	2796
106	3	3	35	271·54	1·93	0·74	92·00	29·025	2959
107	3	3	42	271·96	1·93	0·61	91·31	29·050	3286
108	3	3	38	271·62	2·00	0·72	93·70	29·070	3420
109	3	3	36	271·30	1·98	0·74	94·09	29·085	3124
110	3	3	48	271·31	2·08	0·58	94·11	29·070	3114
111	3	3	46	271·35	1·99	0·58	94·08	29·080	3142
112	3	3	38	271·05	1·90	0·68	96·49	29·110	3382

Pendulum 1821. Eighth Series.

No. of Swing.	Number of Coincidences.		Number of Intervals.	Mean Interval.	Arc of Vibration.		Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rate of Pendulum 1821 upon SHELTON.
	Beginning.	End.			Beginning.	End.			
				s			°	in.	
113	3	3	44	281·12	1·85	0·57	43·05	29·745	9·99693913
114	3	3	46	280·83	1·76	0·53	43·80	29·800	3731
115	3	3	45	280·93	1·95	0·55	43·80	29·880	3891
116	3	3	36	281·16	1·75	0·60	42·26	29·975	3824
117	3	3	50	281·15	1·84	0·48	41·98	30·075	3759
118	3	4	36	281·09	2·08	0·80	41·68	30·190	3780
119	3	3	31	281·31	1·83	0·76	41·46	30·265	3928
120	3	3	46	281·36	2·20	0·60	40·88	30·330	3902
121	3	3	54	281·46	2·14	0·55	40·44	30·380	3906
122	3	3	40	281·65	1·85	0·67	40·54	30·420	4123

Pendulum 8. Fifth Series.

No. of Swing.	Number of Coincidences.		Number of Intervals.	Mean Interval.	Arc of Vibration.		Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rate of Pendulum 8 upon EARNSHAW.
	Beginning.	End.			Beginning.	End.			
				s			°	in.	
83	3	3	29	277·91	1·70	0·75	47·80	29·775	9·99691320
84	3	3	46	277·88	1·96	0·56	47·81	29·750	1281
85	3	3	43	277·91	1·94	0·59	47·80	29·705	1310
86	3	3	44	277·91	2·01	0·60	47·68	29·645	1289
87	3	3	31	277·83	2·11	0·90	47·41	29·605	1246
88	3	3	37	277·91	1·96	0·72	47·28	29·585	1216
89	3	3	40	277·91	1·86	0·60	48·08	29·525	1319
90	3	3	44	277·86	2·04	0·62	48·18	29·460	1314
91	3	3	43	277·88	2·14	0·67	48·05	29·435	1337
92	3	3	33	277·97	1·79	0·72	47·96	29·430	1368

Pendulum 8. Sixth Series.

93	3	3	37	269·56	2·21	0·71	89·69	29·570	9·99690259
94	3	3	48	270·15	1·97	0·58	88·80	29·545	0699
95	3	3	41	270·38	2·21	0·74	84·45	29·535	0168
96	3	3	38	270·39	1·99	0·72	86·71	29·515	0591
97	3	3	47	269·77	2·06	0·60	91·09	29·465	0722
98	3	3	32	270·43	2·17	0·89	90·00	29·430	1394
99	3	3	36	270·82	1·90	0·71	86·78	29·400	1079
100	3	3	47	270·39	2·05	0·61	90·14	29·370	1255
101	3	3	45	270·08	2·14	0·68	89·19	29·355	0721
102	3	3	39	271·21	2·03	0·76	85·26	29·310	1253

Pendulum 8. Seventh Series.

103	3	3	26	278·20	1·94	0·68	52·24	28·970	9·99692447
104	3	3	47	277·73	1·99	0·58	52·45	28·985	1945
105	3	3	48	277·71	2·18	0·67	51·91	29·010	1876
106	3	3	33	277·76	2·15	0·89	51·03	29·025	1818
107	3	3	41	277·94	1·93	0·69	50·79	29·050	1870
108	3	3	37	277·91	2·23	0·86	50·06	29·070	1803
109	3	3	40	278·25	1·93	0·73	48·83	29·085	1832
110	3	3	48	278·32	2·32	0·69	48·61	29·070	1930
111	4	3	40	278·36	2·08	0·76	48·71	29·080	1967
112	3	3	36	278·53	1·94	0·80	48·23	29·110	2052

Pendulum 8. Eighth Series.

No. of Swing.	Number of Coincidences.		Number of Intervals.	Mean Interval.	Arc of Vibration.		Mean of Thermometer Readings.	Mean of Barometer Readings.	Corrected Log. Rate of Pendulum 8 upon EARNSHAW.
	Beginning.	End.			Beginning.	End.			
				s			°	in.	
113	3	3	45	270·14	2·20	0·68	94·28	29·745	9·99691921
114	3	3	47	270·57	2·02	0·61	92·44	29·800	2012
115	3	3	42	270·21	2·08	0·72	93·11	29·880	1775
116	3	3	37	270·51	1·92	0·74	94·44	29·975	2399
117	3	3	53	271·08	1·98	0·50	91·29	30·075	2397
118	3	3	36	270·93	2·10	0·83	93·39	30·190	2787
119	3	3	35	269·97	2·10	0·82	95·80	30·265	2146
120	3	3	49	269·78	2·11	0·58	98·00	30·330	2313
121	3	3	52	270·78	2·07	0·55	95·78	30·380	3047
122	3	3	42	270·90	1·95	0·66	94·39	30·420	2917

SECTION XIII.—*Computation of Log. Rate of Pendulum 1821 on Pendulum 8; combination of individual results; and conclusion on the amount of temperature correction, and on the result of the Harton Experiment.*

72. The quantity $\text{Log.}\frac{\text{Rate of Pendulum 1821}}{\text{Rate of Pendulum 8}}$ is formed, as a similar quantity is formed in Section VI., by combining together

$$\text{Log.}\frac{\text{Rate of Pendulum 1821}}{\text{Rate of SHELTON}}+\text{Log.}\frac{\text{Rate of SHELTON}}{\text{Rate of EARNSHAW}}-\text{Log.}\frac{\text{Rate of Pendulum 8}}{\text{Rate of EARNSHAW}}.$$

Results for the Log. Rate of Pendulum 1821 upon Pendulum 8.

Fifth Series.		Sixth Series.		Seventh Series.		Eighth Series.	
No. of Swing.	$\text{Log.}\frac{\text{Rate Pendulum 1821}}{\text{Rate Pendulum 8}}$.	No. of Swing.	$\text{Log.}\frac{\text{Rate Pendulum 1821}}{\text{Rate Pendulum 8}}$.	No. of Swing.	$\text{Log.}\frac{\text{Rate Pendulum 1821}}{\text{Rate Pendulum 8}}$.	No. of Swing.	$\text{Log.}\frac{\text{Rate Pendulum 1821}}{\text{Rate Pendulum 8}}$.
83	0·00072074	93	0·00072767	103	0·00071958	113	0·00072044
84	71959	94	72205	104	72227	114	71944
85	72363	95	73117	105	72151	115	72562
86	71927	96	73178	106	72380	116	71854
87	72467	97	73015	107	72725	117	71865
88	72667	98	72592	108	72805	118	71526
89	72242	99	72774	109	72363	119	72350
90	72155	100	72538	110	72320	120	72205
91	72123	101	73270	111	72298	121	71687
92	72006	102	72812	112	72240	122	71809

73. We have now to consider how the individual results of each Series are to be combined. And here it is to be remarked, that the principal cause of error is not, in these experiments, the uncertainty of comparisons of clocks (which was almost the only source of error in the experiments at the mine). It is probable that the amount of uncertainty from this cause is here nearly unappreciable. The principal sources of error will lie in the circumstances of the pendulum-observations; chiefly, perhaps,

in the insecurity of the indications of the temperature of the pendulums, as inferred from the readings of the neighbouring thermometers. After consideration of these points, I have thought it best to give equal weights to the results. And for comparison with the simple mean of these, I have taken the simple mean of the thermometer-readings. Thus I obtain,—

	Log. $\frac{\text{Rate Pendulum 1821}}{\text{Rate Pendulum 8}}$.	Temperature of 1821.	Temperature of 8.
Fifth Series	0·00072198	84°·71	47°·80
Sixth Series	0·00072827	53·04	88·21
Seventh Series	0·00072347	93·41	50·29
Eighth Series	0·00071985	41·99	94·29

The interpretation of these numbers is not without difficulty. From the care taken by the observers, it was supposed that the material state of the pendulums would be sensibly the same during the whole continuance of the observations, and it was expected therefore that the results of the Fifth and the Seventh Series would be in close accordance, and that the results of the Sixth and the Eighth Series would be in close accordance. Now the fact is, that the results of the Fifth and Seventh Series agree pretty well, when regard is had to the circumstances of forced temperature (the discordance 0·00000149 corresponds to $0^{s}\cdot3$ per day nearly, or to $0^{\circ}\cdot72$ of temperature); but the discordance between the Sixth and Eighth is 0·00000842 (which corresponds to $1^{s}\cdot68$ per day, or to $4^{\circ}\cdot09$ of temperature). It would therefore seem, at first sight, that there is strong reason to suppose that some change took place in one of the pendulums after the Seventh Series.

74. As there is no reason whatever for supposing a change before the end of the Seventh Series, let us take the mean of the Fifth and Seventh Series, and compare it (A.) with the Sixth Series alone, (B.) with the mean of the Sixth and Eighth Series; and from these comparisons let us in each case obtain a relation between the two pendulums nearly independent of temperature. Thus we shall have,—

Comparison A.

	Log. $\frac{\text{Rate Pendulum 1821}}{\text{Rate Pendulum 8}}$.	Temperature of 1821.	Temperature of 8.
Mean of Fifth and Seventh Series	0·00072273	89°·06	49°·04
Sixth Series	0·00072827	53·04	88·21
Mean of the two lines	0·00072550	71·05	68·62

Comparison B.

Mean of Fifth and Seventh Series	0·00072273	89·06	49·04
Mean of Sixth and Eighth Series	0·00072406	47·52	91·25
Mean of the two lines	0·00072340	68·29	70·15

When it is considered that corrections for temperature have already been applied, founded on the best materials to which at present we have access, it will be perceived that the two "Means of the two lines" are almost strictly independent of temperature. Now we have, in the Harton Experiment, other means of determining the relation between the pendulums. In article 45,

$$\text{Log.}\frac{\text{Rate Pendulum 8 below}}{\text{Rate Pendulum 1821 above}}=9{\cdot}99928558$$

$$\text{Log.}\frac{\text{Rate Pendulum 1821 below}}{\text{Rate Pendulum 8 above}}=0{\cdot}00073694.$$

Subtracting the first from the second, and dividing by 2, the effect of the mine is eliminated, and that of temperature is sensibly eliminated; and we obtain

$$\text{Log.}\frac{\text{Rate Pendulum 1821}}{\text{Rate Pendulum 8}}=0{\cdot}00072568.$$

This agrees so nearly with the result of Comparison A, as greatly to increase the presumption that some change took place in one pendulum after the Seventh Series.

75. Let then z be the increase (in units of the 8th decimal of the logarithm) which ought to be made for every degree of temperature. Taking the difference between the two first lines of Comparison A,

$$554=z\times 75{\cdot}19,$$

whence

$$z=7{\cdot}37 \text{ nearly}$$

$$=\frac{1}{28} \text{ of the correction employed in our tables.}$$

As the mean excess of temperature at the lower station in the Harton Experiment was $7^{\circ}{\cdot}13$, the correction to be added to the rate below is 53, or the correction to the gravity is 106. Therefore (see article 45),

$$\text{Log.}\frac{\text{Gravity below}}{\text{Gravity above}}=0{\cdot}00002358, \text{ or } \frac{\text{Gravity below}}{\text{Gravity above}}=1{\cdot}00005429.$$

Using this number in the equation of article 60,

$$0{\cdot}00006603=0{\cdot}00017984\times\frac{d}{\text{D}},$$

whence $\frac{\text{D}}{d}=2{\cdot}7236$.

And the Earth's mean density (article 62) will result 6·809.

76. If, however, we used Comparison B in the same way, we should have

$$133=z\times 83{\cdot}75,$$

whence

$$z=1{\cdot}59 \text{ nearly},$$

or the alteration to be made in the Earth's mean density is less than one-fourth of that resulting from Comparison A. This gives for the Earth's mean density 6·623 nearly.

Addendum.

On communicating with Professor Stokes, in reference to the effect of the Earth's rotation and ellipticity in modifying the numerical results of the Harton Experiment, I was favoured by that gentleman with an investigation, which, with his permission, I subjoin as a valuable addition to my own paper.

"I shall suppose the surface of the Earth to be an ellipsoid of revolution, and will employ the notation made use of in my paper on Clairaut's Theorem, published in the fourth volume of the Cambridge and Dublin Mathematical Journal. In this,

V is the potential of the Earth's mass.

r, θ are the polar coordinates of any point in or exterior to the Earth's surface; r being measured from the centre, and θ from the axis of rotation.

a is the equatorial radius.

ε the ellipticity.

ω the angular velocity.

m the ratio of the centrifugal force to gravity at the equator.

E the mass of the Earth.

ν the angle between the normal and radius vector at any point of the surface.

In the following investigation, small quantities of the second order are neglected, ε and m being regarded as small quantities of the first order.

If
$$U=V+\frac{\omega^2}{2}r^2\sin^2\theta,$$

the differential coefficients

$$\frac{dU}{dr},\quad \frac{1}{r}\cdot\frac{dU}{d\theta}$$

will give the components of the force along and perpendicular to the radius vector; and, g being the force of gravity,

$$g=-\cos\nu\cdot\frac{dU}{dr}+\sin\nu\cdot\frac{1}{r}\cdot\frac{dU}{d\theta};$$

which becomes, since ν and $\frac{dU}{d\theta}$ are small quantities of the first order,

$$g=-\frac{dU}{dr}.$$

Let v be measured along the vertical; then

$$\frac{dg}{dv}=\cos\nu\cdot\frac{dg}{dr}-\sin\nu\cdot\frac{1}{r}\cdot\frac{dg}{d\theta},$$

or, to the first order,

$$\frac{dg}{dv}=\frac{dg}{dr}=-\frac{d^2U}{dr^2}.$$

Let c be the depth of the mine; then if $\left(\frac{c}{a}\right)^2$ be neglected, we shall have for the value of the fraction $\frac{\text{gravity below}}{\text{gravity above}}$ (which I will call F), calculated on the supposition that all the attracting mass is internal to both stations,

$$F=1-\frac{c}{g}.\frac{dg}{dv},$$

where, after differentiation, r is to be put equal to the radius vector of the surface, namely $a(1-\varepsilon\cos^2\theta)$. Now the value of V (Article 5 of the paper referred to) is

$$V=\frac{E}{r}-\left(\frac{E\varepsilon}{a}-\frac{1}{2}\omega^2a^2\right)\frac{a^3}{r^3}\left(\cos^2\theta-\frac{1}{3}\right),$$

which is true, independently of any particular hypothesis respecting the distribution of matter in the interior of the Earth; so that

$$U=\frac{E}{r}-\left(\frac{E\varepsilon}{a}-\frac{1}{2}\omega^2a^2\right)\frac{a^3}{r^3}\left(\cos^2\theta-\frac{1}{3}\right)+\frac{\omega^2}{2}r^2\sin^2\theta$$

and

$$g=-\frac{dU}{dr}$$

$$=\frac{E}{r^2}-3\left(\frac{E\varepsilon}{a}-\frac{1}{2}\omega^2a^2\right)\frac{a^3}{r^4}\left(\cos^2\theta-\frac{1}{3}\right)-\omega^2r\sin^2\theta,$$

whence

$$-\frac{dg}{dv}=-\frac{dg}{dr}$$

$$=\frac{2E}{r^3}-12\left(\frac{E\varepsilon}{a}-\frac{1}{2}\omega^2a^2\right)\frac{a^3}{r^5}\left(\cos^2\theta-\frac{1}{3}\right)+\omega^2\sin^2\theta.$$

Putting now $r=a(1-\varepsilon\cos^2\theta)$, $\omega^2=m\frac{E}{a^3}$, we find

$$g=\frac{E}{a^2}(1+2\varepsilon\cos^2\theta)-\frac{3E}{a^2}\left(\varepsilon-\frac{m}{2}\right)\left(\cos^2\theta-\frac{1}{3}\right)-m\frac{E}{a^2}(1-\cos^2\theta)$$

$$=\frac{E}{a^2}\left\{1+\left(\frac{5m}{2}-\varepsilon\right)\cos^2\theta+\varepsilon-\frac{3m}{2}\right\}$$

$$-\frac{dg}{dv}=\frac{2E}{a^3}(1+3\varepsilon\cos^2\theta)-\frac{12E}{a^3}\left(\varepsilon-\frac{m}{2}\right)\left(\cos^2\theta-\frac{1}{3}\right)+\frac{mE}{a^3}(1-\cos^2\theta)$$

$$=\frac{2E}{a^3}\left\{1+\left(\frac{5m}{2}-3\varepsilon\right)\cos^2\theta+2\varepsilon-\frac{m}{2}\right\}.$$

Whence

$$-\frac{1}{g}.\frac{dg}{dv}=\frac{2}{a}\left\{\begin{array}{l}1+\left(\frac{5m}{2}-3\varepsilon\right)\cos^2\theta+2\varepsilon-\frac{m}{2}\\ -\left(\frac{5m}{2}-\varepsilon\right)\cos^2\theta-\varepsilon+\frac{3m}{2}\end{array}\right\}$$

$$=\frac{2}{a}\{1-2\varepsilon\cos^2\theta+\varepsilon+m\};$$

and therefore

$$F=1+\frac{2c}{a}\{1-2\varepsilon\cos^2\theta+\varepsilon+m\}.$$

Now the method adopted in the 'Account of Experiments,' &c., article 57, gives

$$-\frac{1}{g}\cdot\frac{dg}{dr}=\frac{2c}{r}=\frac{2c}{a}(1+\varepsilon\cos^2\theta),$$

whence

$$F=1+\frac{2c}{a}(1+\varepsilon\cos^2\theta).$$

Therefore, if R be the ratio of the value of F−1 given above, to F−1 as calculated by the method of the 'Account of Experiments,'

$$R=\frac{1-2\varepsilon\cos^2\theta+\varepsilon+m}{1+\varepsilon\cos^2\theta}=1-3\varepsilon\cos^2\theta+\varepsilon+m.$$

If l be the geocentric latitude of the place, we may in the small term replace θ by $90°-l$; and since $\cos^2\theta=\sin^2 l=\frac{1}{2}(1-\cos 2l)$, we find

$$R=1+m-\frac{\varepsilon}{2}+\frac{3\varepsilon}{2}\cos 2l.$$

Now

$$m=\frac{1}{289}=0\cdot00346$$

$$\varepsilon=\frac{1}{300\cdot8}=0\cdot00333$$

l, for Harton, $=54°\ 48'$;

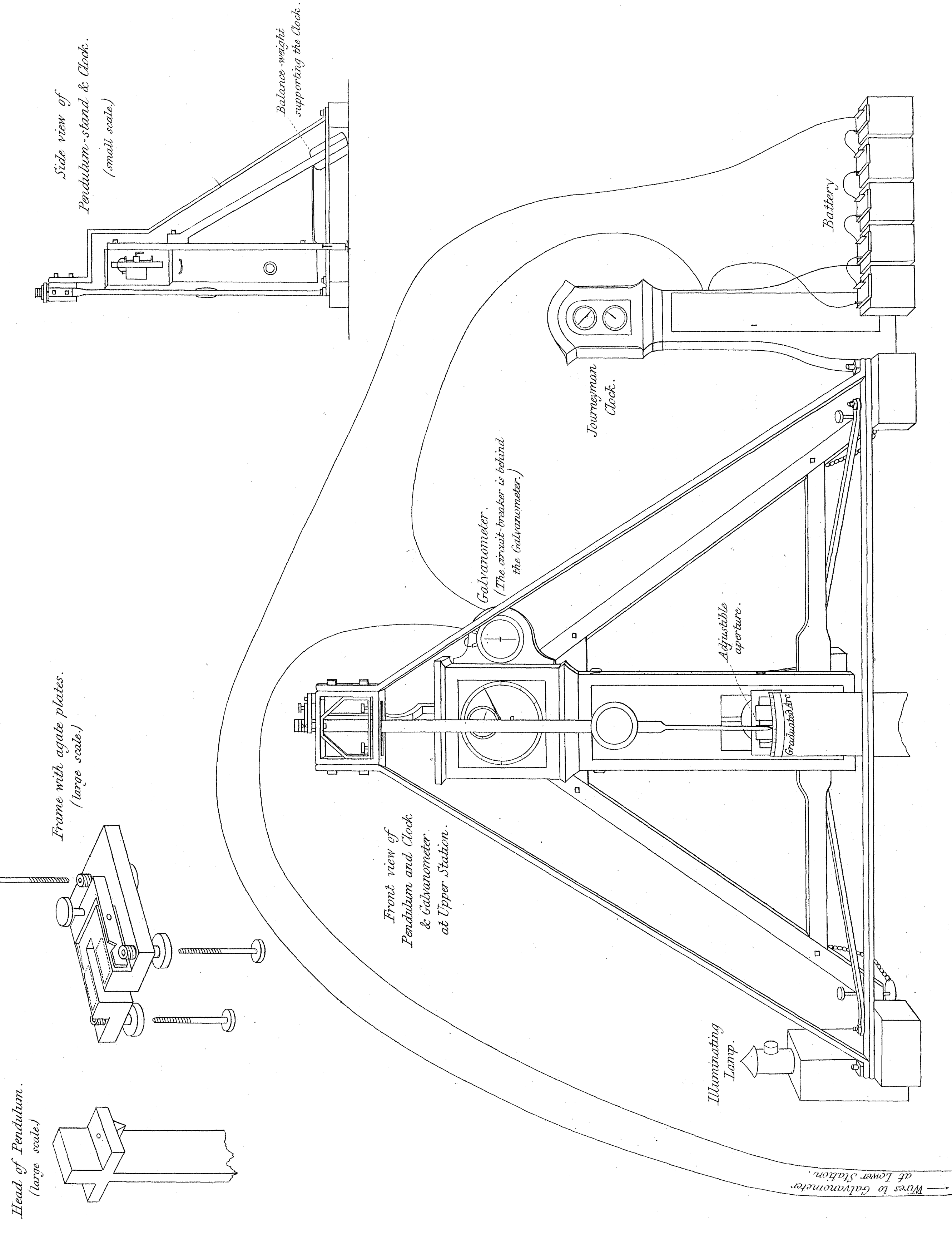
$$R=1+0\cdot00346-0\cdot00334=1\cdot00012.$$

That R should have been so very nearly equal to unity, depends upon an accidental numerical relation between the values of m, ε, and l. At the equator, R−1 would have been as great as 0·00679.

In article 60 of the 'Account,' F−1 was found =·00012032; whence R.(F−1)=·00012033; which only alters the final value of the mean density in the ratio of 6836 to 6835, giving for result

6·565.

At the equator, the correction to the deduced value 6·566 would have been −·077."


Side view of Pendulum-stand & Clock.
(small scale.)
Balance-weight supporting the Clock.
Battery
Journeyman Clock.
Galvanometer.
(The circuit-breaker is behind the Galvanometer.)
Adjustible aperture.
Graduated Arc
Front view of Pendulum and Clock & Galvanometer at Upper Station.
Frame with agate plates.
(large scale.)
Illuminating Lamp.
Head of Pendulum.
(large scale.)
Wires to Galvanometer at Lower Station.

J. Basire sc.

Fig. 1.

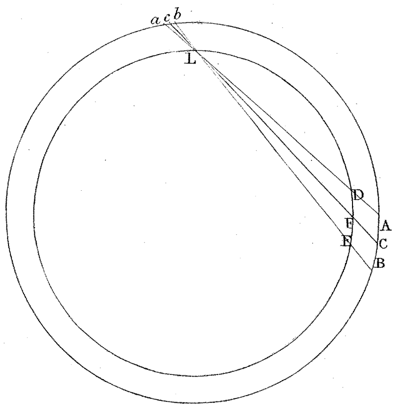


Fig. 2.

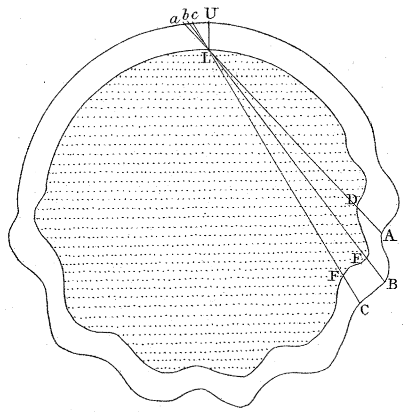
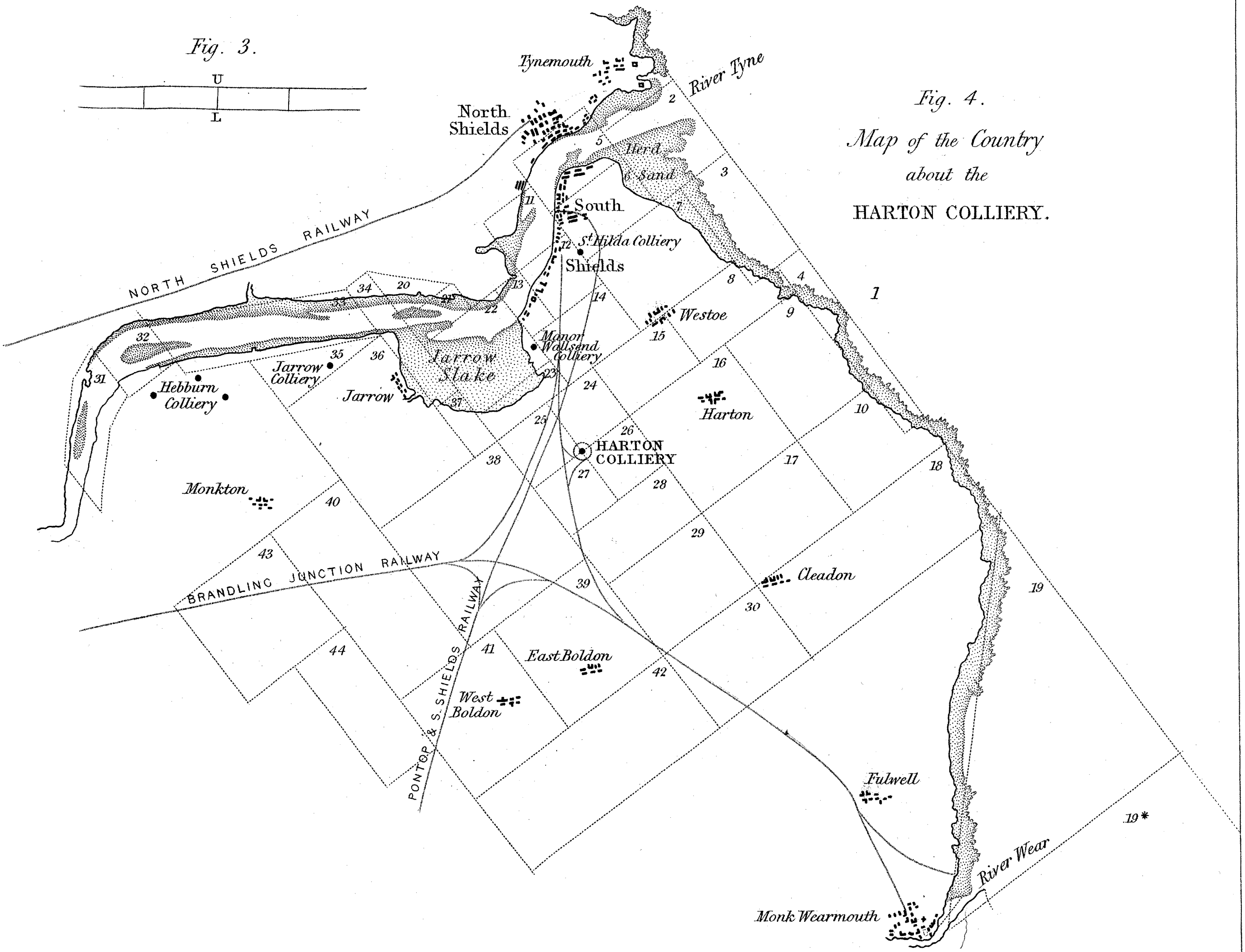


Fig. 3.

Fig. 4.

Map of the Country about the HARTON COLLIERY.



Scale of Depths.